PEARSON TEXAS ALGEBRA I

STUDENT TEXT AND HOMEWORK HELPER

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TOPIC 5 Exponents and Radicals

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

(11)(A) Simplify numerical radical expressions involving square roots.

(11)(B) Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

TOPIC 6 Sequences

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

(12)(B) Evaluate functions, expressed in function notation, given one or more elements in their domains.

(12)(C) Identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

(12)(D) Write a formula for the nth term of arithmetic and geometric sequences, given the value of several of their terms.

TOPIC 7 Polynomials and Factoring

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

(10)(A) Add and subtract polynomials of degree one and degree two.

(10)(B) Multiply polynomials of degree one and degree two.

(10)(C) Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend.

(10)(D) Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

(10)(E) Factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

(10)(F) Decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

Topic 5 | Exponents and Radicals

TOPIC OVERVIEW

5-1 Zero and Negative Exponents
5-2 Multiplying Powers With the Same Base
5-3 More Multiplication Properties of Exponents
5-4 Division Properties of Exponents
5-5 Rational Exponents and Radicals
5-6 Simplifying Radicals
5-7 The Pythagorean Theorem

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
hypotenuse, *p. 233*	hipotenusa
index, *p. 225*	índice
leg, *p. 233*	cateto
principal square root, *p. 225*	raíz cuadrada principal
Pythagorean Theorem, *p. 233*	Teorema de Pitágoras
radical, *p. 225*	radical
radical expression, *p. 225*	expressión radical
radicand, *p. 225*	radicando
rationalize the denominator, *p. 229*	racionalizar el denominador
square root, *p. 225*	raíz cuadrada

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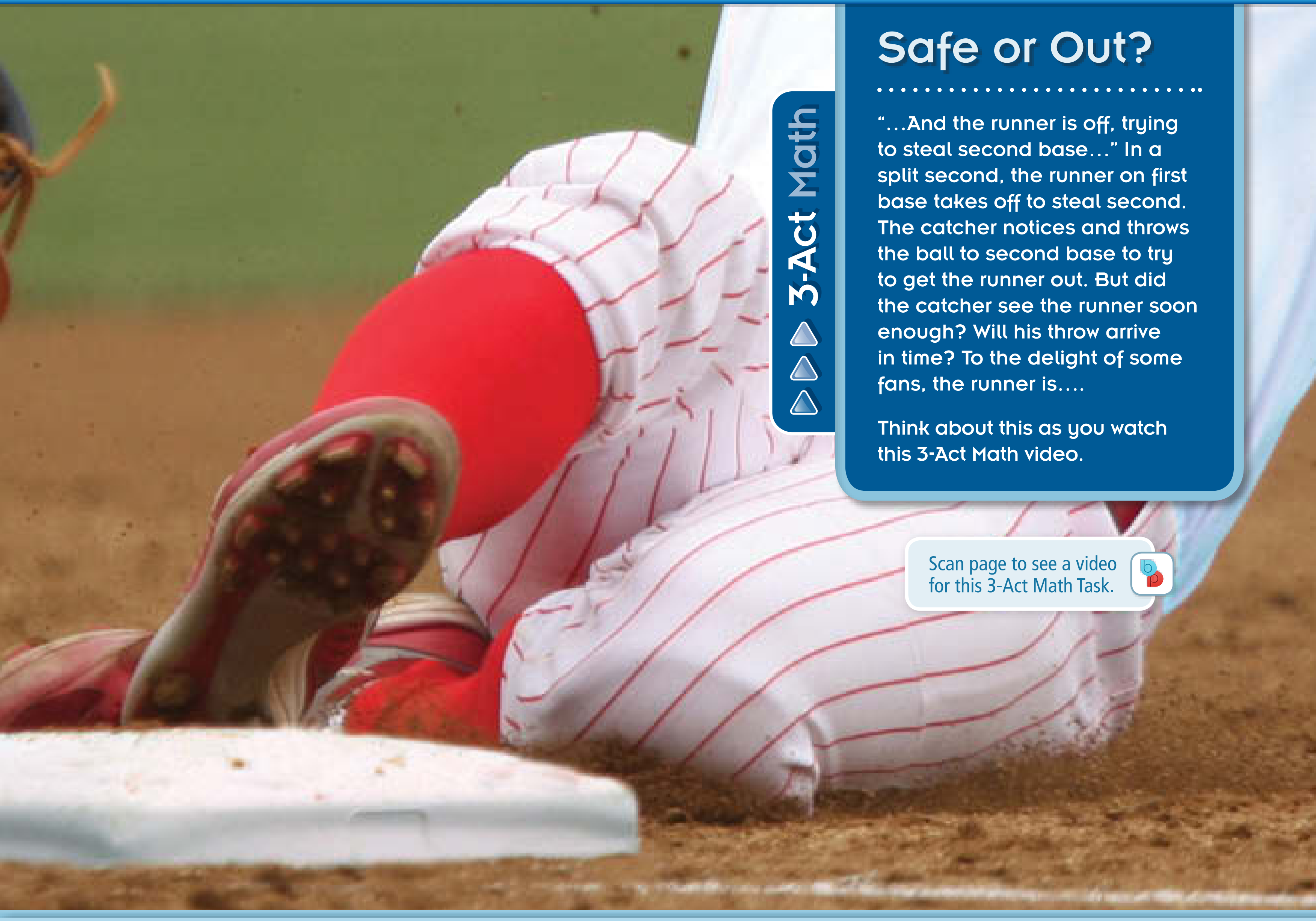

3-Act Math

Safe or Out?

"…And the runner is off, trying to steal second base…" In a split second, the runner on first base takes off to steal second. The catcher notices and throws the ball to second base to try to get the runner out. But did the catcher see the runner soon enough? Will his throw arrive in time? To the delight of some fans, the runner is….

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5-1 Zero and Negative Exponents

TEKS FOCUS

TEKS (11)(B) Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(D)

VOCABULARY

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

You can extend the idea of exponents to include zero and negative exponents.

take note — Properties Zero and Negative Exponents

Zero as an Exponent For every nonzero number a, $a^0 = 1$.

Examples $4^0 = 1$ $\quad (-3)^0 = 1$ $\quad (5.14)^0 = 1$

Negative Exponent For every nonzero number a and integer n, $a^{-n} = \frac{1}{a^n}$.

Examples $7^{-3} = \frac{1}{7^3}$ $\quad (-5)^{-2} = \frac{1}{(-5)^2}$

Problem 1

TEKS Process Standard (1)(E)

Think

Can you use the definition of zero as an exponent when the base is a negative number?
Yes, the definition of zero as an exponent is true for all nonzero bases.

Simplifying Powers

What is the simplified form of each expression?

A 9^{-2}

$9^{-2} = \frac{1}{9^2}$ Use the definition of negative exponent.

$= \frac{1}{81}$ Simplify.

B $(-3.6)^0$

$(-3.6)^0 = 1$ Use the definition of zero as an exponent.

Problem 2

Think

Which part of the expression do you need to rewrite?
The base b has a negative exponent; rewrite it with a positive exponent.

Simplifying Exponential Expressions

What is the simplified form of each expression?

A $5a^3b^{-2}$

$5a^3b^{-2} = 5a^3\left(\frac{1}{b^2}\right)$ Use the definition of negative exponent.

$= \frac{5a^3}{b^2}$ Simplify.

continued on next page ▶

Problem 2 *continued*

B $\frac{1}{x^{-5}}$

$\frac{1}{x^{-5}} = 1 \div x^{-5}$ Rewrite using a division symbol.

$= 1 \div \frac{1}{x^5}$ Use the definition of negative exponent.

$= 1 \cdot x^5$ Multiply by the reciprocal of $\frac{1}{x^5}$, which is x^5.

$= x^5$ Identity Property of Multiplication

Problem 3

Evaluating an Exponential Expression

Plan

How do you simplify the expression?
Use the definition of negative exponent to rewrite the expression with only positive exponents.

What is the value of $3s^3t^{-2}$ for $s = 2$ and $t = -3$?

Method 1 Simplify first.

$$3s^3t^{-2} = \frac{3(s)^3}{t^2} = \frac{3(2)^3}{(-3)^2} = \frac{24}{9} = 2\frac{2}{3}$$

Method 2 Substitute first.

$$3s^3t^{-2} = 3(2)^3(-3)^{-2} = \frac{3(2)^3}{(-3)^2} = \frac{24}{9} = 2\frac{2}{3}$$

Problem 4

TEKS Process Standard (1)(D)

Using an Exponential Expression STEM

Population Growth **A population of marine bacteria doubles every hour under controlled laboratory conditions. The number of bacteria is modeled by the expression $1000 \cdot 2^h$, where h is the number of hours after a scientist measures the population size. Evaluate the expression for $h = 0$ and $h = -3$. What does each value of the expression represent in the situation?**

Know
$1000 \cdot 2^h$ models the population.

Need
Values of the expression for $h = 0$ and $h = -3$

Plan
Substitute each value of h into the expression and simplify.

$1000 \cdot 2^h = 1000 \cdot 2^0$ Substitute 0 for h.

$= 1000 \cdot 1 = 1000$ Simplify.

The value of the expression for $h = 0$ is 1000. There were 1000 bacteria at the time the scientist measured the population.

$1000 \cdot 2^h = 1000 \cdot 2^{-3}$ Substitute -3 for h.

$= 1000 \cdot \frac{1}{8} = 125$ Simplify.

The value of the expression for $h = -3$ is 125. There were 125 bacteria 3 h before the scientist measured the population.

PRACTICE and APPLICATION EXERCISES

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Simplify each expression.

1. 3^{-2} **2.** $(-4.25)^0$ **3.** $(-5)^{-2}$ **4.** -5^{-2}

5. 2^{-6} **6.** -3^0 **7.** -12^{-1} **8.** $\frac{1}{2^0}$

9. Apply Mathematics (1)(A) The number of visitors to a certain web site triples every month. The number of visitors is modeled by the expression $8100 \cdot 3^m$, where m is the number of months after the number of visitors was measured. Evaluate the expression for $m = -4$. What does the value of the expression represent in the situation?

STEM **10. Apply Mathematics (1)(A)** A Galápagos cactus finch population increases by half every decade. The number of finches is modeled by the expression $45 \cdot 1.5^d$, where d is the number of decades after the population was measured. Evaluate the expression for $d = -2$, $d = 0$, and $d = 1$. What does each value of the expression represent in the situation?


Galápagos cactus finch

Analyze Mathematical Relationships (1)(F) Is the value of each expression *positive* or *negative*?

11. -2^2 **12.** $(-2)^2$

13. $(-2)^3$ **14.** $(-2)^{-3}$

Write each number as a power of 10 using negative exponents.

15. $\frac{1}{10}$ **16.** $\frac{1}{100}$

17. $\frac{1}{1000}$ **18.** $\frac{1}{10{,}000}$

19. a. Analyze Mathematical Relationships (1)(F) Complete the pattern using powers of 5.

$\frac{1}{5^2} = \blacksquare$ $\frac{1}{5^1} = \blacksquare$ $\frac{1}{5^0} = \blacksquare$ $\frac{1}{5^{-1}} = \blacksquare$ $\frac{1}{5^{-2}} = \blacksquare$

b. Write $\frac{1}{5^{-4}}$ using a positive exponent.

c. Rewrite $\frac{1}{a^{-n}}$ as a power of a.

Rewrite each fraction with all the variables in the numerator.

20. $\frac{a}{b^{-2}}$ **21.** $\frac{4g}{h^3}$ **22.** $\frac{5m^6}{3n}$ **23.** $\frac{8c^5}{11d^4e^{-2}}$

24. Use Multiple Representations to Communicate Mathematical Ideas (1)(D) Suppose your drama club's budget doubles every year. This year the budget is \$500. How much was the club's budget 2 yr ago?

25. Copy and complete the table at the right.

n	3	■	■	$\frac{5}{8}$	■
n^{-1}	■	6	$\frac{1}{7}$	■	0.5

26. a. Simplify $a^n \cdot a^{-n}$.

b. Explain Mathematical Ideas (1)(G) What is the mathematical relationship between a^n and a^{-n}? Explain.

Simplify each expression.

27. $4ab^0$ **28.** $\frac{1}{x^{-7}}$ **29.** $\frac{3^{-2}}{n}$ **30.** $k^{-4}j^0$

31. $\frac{7ab^{-2}}{3w}$ **32.** $c^{-5}d^{-7}$ **33.** $c^{-5}d^7$ **34.** $\frac{8}{2s^{-3}}$

Evaluate each expression for $r = -3$ and $s = 5$.

35. r^{-3} **36.** s^{-3} **37.** $\frac{3r}{s^{-2}}$ **38.** $\frac{s^0}{r^{-2}}$

39. $4s^{-1}$ **40.** r^0s^{-2} **41.** $r^{-4}s^2$ **42.** $2^{-4}r^3s^{-2}$

43. Create Representations to Communicate Mathematical Ideas (1)(E) Choose a fraction to use as a value for the variable a. Find the values of a^{-1}, a^2, and a^{-2}.

STEM **44. Apply Mathematics (1)(A)** A company is making metal rods with a target diameter of 1.5 mm. A rod is acceptable when its diameter is within 10^{-3} mm of the target diameter. Write an inequality for the acceptable range of diameters.

45. Justify Mathematical Arguments (1)(G) Are $3x^{-2}$ and $3x^2$ reciprocals? Explain.

Simplify each expression.

46. $\left(\frac{r^{-7}b^{-8}}{t^{-4}w^1}\right)^0$ **47.** $(-5)^2 - (0.5)^{-2}$ **48.** $\frac{6}{m^2} + \frac{5m^{-2}}{3^{-3}}$

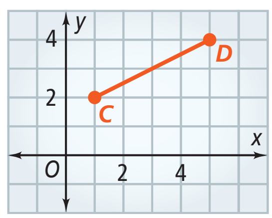
49. $2^3(5^0 - 6m^2)$ **50.** $\frac{2x^{-5}y^3}{n^2} \div \frac{r^2y^5}{2n}$ **51.** $2^{-1} - \frac{1}{3^{-2}} + 5\left(\frac{1}{2^2}\right)$

52. For what value or values of n is $n^{-3} = \left(\frac{1}{n}\right)^5$?

TEXAS End-of-Course PRACTICE

53. What is the simplified form of $-6(-6)^{-1}$?

54. Segment CD represents the flight of a bird that passes through the points (1, 2) and (5, 4). What is the slope of a line that represents the flight of a second bird that flew perpendicular to the first bird?



55. What is the solution of the equation $1.5(x - 2.5) = 3$?

56. What is the simplified form of $|3.5 - 4.7| + 5.6$?

57. What is the y-intercept of the graph of $3x - 2y = -8$?

5-2 Multiplying Powers With the Same Base

TEKS FOCUS

TEKS (11)(B) Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

TEKS (1)(A) **Apply** mathematics to problems arising in everyday life, society, and the workplace.

Additional TEKS (1)(E)

VOCABULARY

- **Apply** – use knowledge or information for a specific purpose, such as solving a problem

ESSENTIAL UNDERSTANDING

You can use a property of exponents to multiply powers with the same base.

take note — Property Multiplying Powers With the Same Base

Words To multiply powers with the same base, add the exponents.

Algebra $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$ and m and n are rational numbers.

Examples $b^7 \cdot b^{-4} = b^{7+(-4)} = b^3$ $\quad$ $4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} = 4^{\frac{1}{3}+\frac{1}{3}} = 4^{\frac{2}{3}}$

take note — Key Concept Using Rational Exponents

Exponents can also be expressed as fractions. Fractional exponents are called *rational exponents*.

Recall that 3^2 means $3 \cdot 3$, which equals 9. You can represent the number 3 using a rational exponent: $9^{\frac{1}{2}} = 3$. The equation $9^{\frac{1}{2}} = b$ indicates that b is the positive number that when multiplied by itself, equals 9.

$$9^{\frac{1}{2}} = 3 \text{ since } 3 \cdot 3 = 9.$$

In general, $a^{\frac{1}{n}} = b$ means that b multiplied as a factor n times equals a.

The definition of a negative exponent stated for integer exponents in Lesson 5-1 also holds for rational exponents.

Problem 1

TEKS Process Standard (1)(E)

Think

When can you multiply powers? You can use the property for multiplying powers when the bases of the powers are the same.

Multiplying Powers

What is each expression written using each base only once?

A $12^4 \cdot 12^3 = 12^{4+3}$ Add the exponents of the powers with the same base.

$= 12^7$ Simplify the exponent.

continued on next page ▶

Problem 1 *continued*

B $(-5)^{-2}(-5)^{7} = (-5)^{-2+7}$ — Add the exponents of the powers with the same base.

$= (-5)^{5}$ — Simplify the exponent.

Problem 2

Multiplying Powers in Algebraic Expressions

Plan

Which parts of the expression can you combine?
You can group the coefficients and multiply. You can also write any powers that have the same base with a single exponent.

What is the simplified form of each expression?

A $4z^5 \cdot 9z^{-12} = (4 \cdot 9)\left(z^5 \cdot z^{-12}\right)$ — Commutative and associative properties of multiplication

$= 36\left(z^{5+(-12)}\right)$ — Multiply the coefficients. Add the exponents of the powers with the same base.

$= 36z^{-7}$ — Simplify the exponent.

$= \dfrac{36}{z^7}$ — Rewrite using a positive exponent.

B $2a \cdot 9b^4 \cdot 3a^2 = (2 \cdot 9 \cdot 3)\left(a \cdot a^2\right)\left(b^4\right)$ — Commutative and associative properties of multiplication

$a = a^1$

$= 54\left(a^1 \cdot a^2\right)\left(b^4\right)$ — Multiply the coefficients. Write a as a^1.

$= 54\left(a^{1+2}\right)\left(b^4\right)$ — Add exponents of powers with the same base.

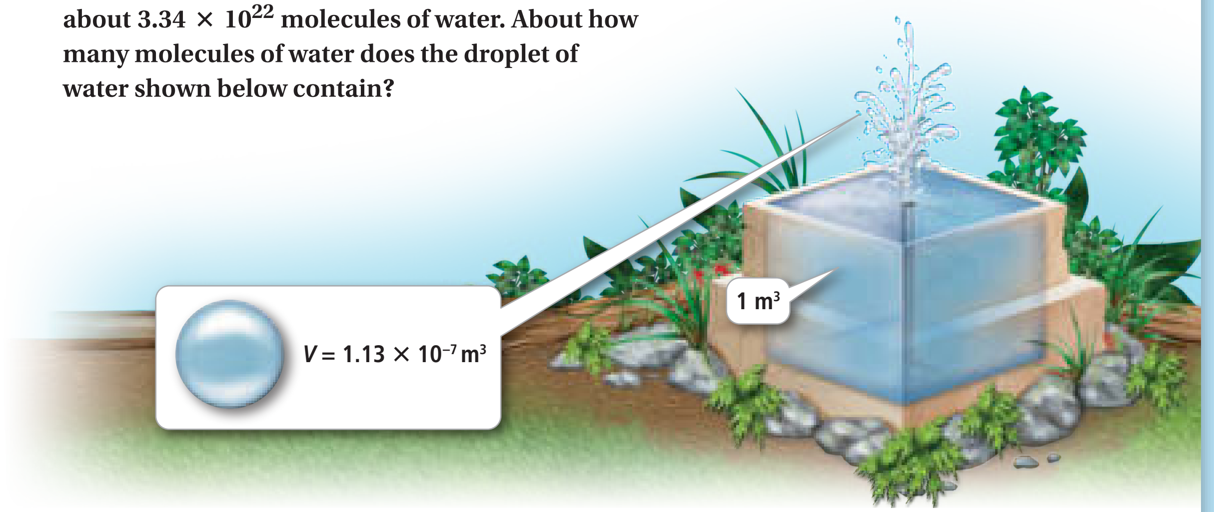
$= 54a^3b^4$ — Simplify.

Problem 3

TEKS Process Standard (1)(A)

Multiplying With Scientific Notation STEM

Chemistry **At 20°C, one cubic meter of water has a mass of about 9.98×10^5 g. Each gram of water contains about 3.34×10^{22} molecules of water. About how many molecules of water does the droplet of water shown below contain?**



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Problem 3 *continued*

Plan

How do you find the number of molecules?
Use unit analysis. Divide out the common units.

$$\text{molecules of water} = \cancel{\text{cubic meters}} \cdot \frac{\cancel{\text{grams}}}{\cancel{\text{cubic meters}}} \cdot \frac{\text{molecules}}{\cancel{\text{grams}}}$$ Use unit analysis.

$$= (1.13 \times 10^{-7}) \cdot (9.98 \times 10^{5}) \cdot (3.34 \times 10^{22})$$ Substitute.

$$= (1.13 \cdot 9.98 \cdot 3.34) \times (10^{-7} \cdot 10^{5} \cdot 10^{22})$$ Commutative and associative properties of multiplication

$$\approx 37.7 \times 10^{-7+5+22}$$ Multiply. Add exponents.

$$= 37.7 \times 10^{20}$$ Simplify.

$$= 3.77 \times 10^{21}$$ Write in scientific notation.

The droplet contains about 3.77×10^{21} molecules of water.

Problem 4

Think

What number multiplied by itself 4 times equals 81?
$9 \cdot 9 = 81$ and $(3 \cdot 3)(3 \cdot 3) = 81$.

Simplifying Expressions With Rational Exponents

Simplify the expression $81^{\frac{1}{4}}$.

$81^{\frac{1}{4}}$ Find the number that when multiplied by itself four times gives 81.

$81^{\frac{1}{4}} = 3$ $\quad 3 \cdot 3 \cdot 3 \cdot 3 = 81$

Problem 5

Think

How can the fractional exponent be rewritten?
Exponents can be rewritten as multiple factors if the base of each exponential factor is the same.

Simplifying Expressions With Rational Exponents

Simplify the expression $64^{\frac{3}{2}}$.

$$64^{\frac{3}{2}} = 64^{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}$$ Rewrite the fractional exponent as a sum of fractions of the form $\frac{1}{n}$.

$$= 64^{\frac{1}{2}} \cdot 64^{\frac{1}{2}} \cdot 64^{\frac{1}{2}}$$ Use the Property of Multiplying Powers With the Same Base.

$$= 8 \cdot 8 \cdot 8$$ Since $8 \cdot 8 = 64$, substitute 8 for $64^{\frac{1}{2}}$.

$$= 512$$ Simplify.

Problem 6

Think

Why must like variables be grouped together?
To simplify by adding exponents, the bases must be the same.

Simplifying Expressions With Rational Exponents

Simplify the expression $\left(2a^{\frac{2}{3}} \cdot 3b^{\frac{1}{4}}\right)\left(a^{\frac{1}{3}} \cdot 5b^{\frac{1}{2}}\right)$.

$$= (2 \cdot 3 \cdot 5)\left(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}}\right)\left(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}\right)$$ Commutative and associative properties of multiplication

$$= 30\left(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}}\right)\left(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}\right)$$ Simplify.

$$= 30\left(a^{\frac{3}{3}}\right)\left(b^{\frac{3}{4}}\right)$$ Add exponents that have the same base.

$$= 30ab^{\frac{3}{4}}$$ Simplify.

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Rewrite each expression using each base only once.

1. $7^3 \cdot 7^4$
2. $5^{-2} \cdot 5^{-4} \cdot 5^8$
3. $(-8)^5 \cdot (-8)^{-5}$

Simplify each expression.

4. $m^3 m^4$
5. $\left(5x^5\right)\left(3y^6\right)\left(3x^2\right)$
6. $-m^2 \cdot 4r^3 \cdot 12r^{-4} \cdot 5m$

Write each answer in scientific notation.

STEM 7. **Apply Mathematics (1)(A)** A human body contains about 2.7×10^4 microliters (μL) of blood for each pound of body weight. Each microliter of blood contains about 7×10^4 white blood cells. About how many white blood cells are in the body of a 140-lb person?

STEM 8. **Apply Mathematics (1)(A)** The distance light travels in one second (one light-second) is about 1.86×10^5 mi. Saturn is about 4750 light-seconds from the sun. About how many miles from the sun is Saturn?

Simplify each expression.

9. $8^{\frac{1}{3}}$
10. $625^{\frac{1}{4}}$
11. $16^{\frac{3}{4}}$
12. $9^{\frac{5}{2}}$
13. $\left(7d^{\frac{3}{2}} \cdot 2g^{\frac{5}{6}}\right)\left(2g^{\frac{3}{2}} \cdot 7d^{\frac{5}{6}}\right)$
14. $\left(4r^{\frac{2}{5}} \cdot 5s^{\frac{2}{7}}\right)\left(5s^{\frac{5}{7}} \cdot 4r^{\frac{3}{5}}\right)$

15. **Apply Mathematics (1)(A)** A liter of water contains about 3.35×10^{25} molecules. The Mississippi River discharges about 1.7×10^7 L of water every second. About how many molecules does the Mississippi River discharge every minute? Write your answer in scientific notation.

Complete each equation.

16. $5^2 \cdot 5^{\blacksquare} = 5^{11}$
17. $a^{\frac{2}{3}} \cdot a^{\blacksquare} = a^{\frac{5}{6}}$
18. $x^3 y^{\blacksquare} \cdot x^{\blacksquare} = y^2$

19. When you simplify an algebraic expression like $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$, you know that the bases of the expressions must be the same. You also need to rewrite the exponents so that they have a common denominator.
    a. Explain why you need to find the common denominator to simplify.
    b. Simplify the expression $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$.

Simplify each expression. Write each answer in scientific notation.

20. $\left(9 \times 10^7\right)\left(3 \times 10^{-16}\right)$
21. $\left(0.5 \times 10^{-6}\right)\left(0.3 \times 10^{-2}\right)$
22. $\left(0.2 \times 10^5\right)\left(4 \times 10^{-12}\right)$

STEM 23. **Apply Mathematics (1)(A)** In chemistry, a *mole* is a unit of measure equal to 6.02×10^{23} atoms of a substance. The mass of a single neon atom is about 3.35×10^{-23} g. What is the mass of 2 moles of neon atoms? Write your answer in scientific notation.

Simplify each expression.

24. $\dfrac{1}{a^4 \cdot a^{-3}}$ **25.** $8m^{\frac{1}{3}}\left(m^{\frac{1}{3}}+2\right)$ **26.** $-4x^3\left(3x^3-10x\right)$

27. **Apply Mathematics (1)(A)** A book shows an enlarged photo of a carpenter bee. A carpenter bee is about 6×10^{-3} m long. The photo is 13.5 cm long. About how many times as long as a carpenter bee is the photo?

28. a. **Create Representations to Communicate Mathematical Ideas (1)(E)** Write y^6 as a product of two powers with the same base in four different ways. Use only positive exponents.

b. Write y^6 as a product of two powers with the same base in four different ways, using negative or zero exponents in each product.

c. **Explain Mathematical Ideas (1)(G)** How many ways can you write y^6 as the product of two powers? Explain your reasoning.

Simplify each expression.

29. $3^x \cdot 3^{2-x} \cdot 3^2$ **30.** $3^{\frac{1}{4}} \cdot 2^y \cdot 3^2 \cdot 2^x$ **31.** $(t+3)^{\frac{4}{5}}(t+3)^{\frac{2}{5}}$

TEXAS End-of-Course PRACTICE

32. What is the simplified form of $\left(2x^{\frac{1}{2}}y^{\frac{2}{3}}\right)\left(4x^{\frac{1}{4}}y^{\frac{5}{6}}\right)$?

A. $6x^{\frac{1}{2}}y^{\frac{2}{3}}$ **B.** $6xy$ **C.** $8x^{\frac{1}{2}}y^{\frac{7}{9}}$ **D.** $8x^{\frac{3}{4}}y^{\frac{3}{2}}$

33. What is the x-intercept of the graph of $5x - 3y = 30$?

F. -10 **G.** -6 **H.** 6 **J.** 10

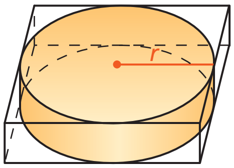
34. At the Athens Olympics, the winning time for the women's 100-m hurdles was 2.06×10^{-1} min. Which number is another way to express this time in minutes?

A. 0.206 **B.** 20.6 **C.** 206×10^1 **D.** 206×10^{-2}

35. What is the solution of $4x - 5 = 2x + 13$?

F. 3 **G.** 4 **H.** 9 **J.** 32

36. Bill's company packages its circular mirrors in boxes with square bottoms, as shown at the right. Show your work for each answer.



a. What is an expression for the area of the bottom of the box?

b. If the mirror has a radius of 4 in., what is the area of the bottom of the box?

c. The area of the bottom of a second box is 196 in.^{2}. What is the diameter of the largest mirror the box can hold?

5-3 More Multiplication Properties of Exponents

TEKS FOCUS

TEKS (11)(B) Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(A)

VOCABULARY

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

You can use laws of exponents to simplify a power raised to a power or a product raised to a power.

take note — Property: Raising a Power to a Power

Words To raise a power to a power, multiply the exponents.

Algebra $(a^m)^n = a^{mn}$, where $a \neq 0$ and m and n are rational numbers.

Examples $(5^4)^2 = 5^{4 \cdot 2} = 5^8$ $\quad (m^3)^5 = m^{3 \cdot 5} = m^{15}$

$(a^{\frac{3}{2}})^3 = a^{\frac{3}{2} \cdot 3} = a^{\frac{9}{2}}$ $\quad (x^{\frac{1}{2}})^{\frac{3}{5}} = x^{\frac{1}{2} \cdot \frac{3}{5}} = x^{\frac{3}{10}}$

take note — Property: Raising a Product to a Power

Words To raise a product to a power, raise each factor to the power and multiply.

Algebra $(ab)^n = a^n b^n$, where $a \neq 0$, $b \neq 0$, and n is a rational number.

Examples $(3x)^4 = 3^4 x^4 = 81x^4$ $\quad (4b)^{\frac{3}{2}} = 4^{\frac{3}{2}} b^{\frac{3}{2}} = 8b^{\frac{3}{2}}$

Problem 1

Simplifying a Power Raised to a Power

Think

Should you add or multiply the exponents to simplify the expression?
You multiply the exponents when raising a power to a power.

A **What is the simplified form of $(n^4)^7$?**

$(n^4)^7 = n^{4 \cdot 7}$ Multiply exponents when raising a power to a power.

$= n^{28}$ Simplify.

B **What is the simplified form of $(x^{\frac{2}{3}})^{\frac{1}{2}}$?**

$(x^{\frac{2}{3}})^{\frac{1}{2}} = x^{\frac{2}{3} \cdot \frac{1}{2}}$ Multiply exponents when raising a rational power to a rational power.

$= x^{\frac{2}{6}} = x^{\frac{1}{3}}$ Simplify.

Problem 2

TEKS Process Standard (1)(E)

Simplifying an Expression With Powers

What is the simplified form of $y^3(y^{\frac{5}{2}})^{-2}$?

Plan

What is the first step in simplifying the expression?
By the order of operations, you simplify powers before you multiply.

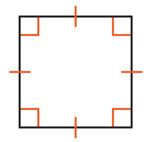
Think	Write
You multiply exponents when raising a power to a power.	$y^3(y^{\frac{5}{2}})^{-2} = y^3y^{\frac{5}{2}\cdot(-2)}$ $= y^3y^{-\frac{10}{2}}$
You add exponents when multiplying powers with the same base.	$= y^{3+(-5)}$ $= y^{-2}$
Write the expression using only positive exponents.	$= \frac{1}{y^2}$

Problem 3

TEKS Process Standard (1)(D)

Simplifying a Product Raised to a Power

Multiple Choice **Which expression represents the area of the square?**



$5x^3$

(A) $10x^3$
(B) $5x^6$
(C) $25x^5$
(D) $25x^6$

Plan

How do you find the area of the square?
The area of a square with side length s is s^2.

$(5x^3)^2 = 5^2(x^3)^2$ Raise each factor to the second power.

$= 5^2x^6$ Multiply the exponents of a power raised to a power.

$= 25x^6$ Simplify.

The correct answer is D.

Problem 4

Simplifying an Expression With Products

What is the simplified form of $(n^{\frac{1}{2}})^{10}(4mn^{-\frac{2}{3}})^3$?

Think

What is the exponent of m?
It has an implied exponent of 1. Similar to coefficients, exponents of 1 don't need to be written.

$(n^{\frac{1}{2}})^{10}(4mn^{-\frac{2}{3}})^3 = (n^{\frac{1}{2}})^{10}4^3m^3(n^{-\frac{2}{3}})^3$ Raise each factor of $4mn^{-\frac{2}{3}}$ to the third power.

$= n^54^3m^3n^{-2}$ Multiply the exponents of a power raised to a power.

$= 4^3m^3n^5n^{-2}$ Commutative Property of Multiplication

$= 4^3m^3n^{5+(-2)}$ Add the exponents of powers with the same base.

$= 64m^3n^3$ Simplify.

Problem 5

Raising a Number in Scientific Notation to a Power STEM

Aircraft **The expression $\frac{1}{2}mv^2$ gives the kinetic energy, in joules, of an object with a mass of m kg traveling at a speed of v meters per second. What is the kinetic energy of an experimental unmanned jet with a mass of 1.3×10^3 kg traveling at a speed of about 3.1×10^3 m/s?**

Plan

How do you raise a number in scientific notation to a power?
A number written in scientific notation is a product. Use the property for raising a product to a power.

$\frac{1}{2}mv^2 = \frac{1}{2} \cdot (1.3 \times 10^3)(3.1 \times 10^3)^2$ — Substitute the values for m and v into the expression.

$= \frac{1}{2} \cdot 1.3 \cdot 10^3 \cdot 3.1^2 \cdot (10^3)^2$ — Raise the two factors to the second power.

$= \frac{1}{2} \cdot 1.3 \cdot 10^3 \cdot 3.1^2 \cdot 10^6$ — Multiply the exponents of a power raised to a power.

$= \frac{1}{2} \cdot 1.3 \cdot 3.1^2 \cdot 10^3 \cdot 10^6$ — Use the Commutative Property of Multiplication.

$= \frac{1}{2} \cdot 1.3 \cdot 3.1^2 \cdot 10^{3+6}$ — Add exponents of powers with the same base.

$= 6.2465 \times 10^9$ — Simplify. Write in scientific notation.

The aircraft has a kinetic energy of about 6.2×10^9 joules.

PRACTICE and APPLICATION EXERCISES

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Simplify each expression.

1. $(n^8)^4$ **2.** $(n^4)^8$ **3.** $(c^2)^{\frac{1}{4}}$ **4.** $(x^{\frac{2}{5}})^{10}$

5. $(a^{\frac{2}{3}})^3c^4$ **6.** $(c^3)^{\frac{1}{9}}(d^3)^0$ **7.** $(t^2)^{-2}(t^2)^{-5}$ **8.** $(m^3)^{-1}(x^{\frac{1}{3}})^{\frac{1}{4}}$

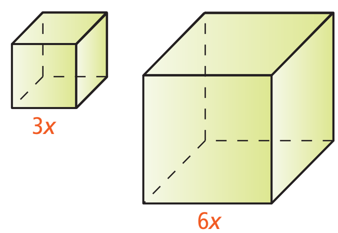
9. The radius of a cylinder is 7.8×10^{-4} m. The height of the cylinder is 3.4×10^{-2} m. What is the volume of the cylinder? Write your answer in scientific notation. (*Hint:* $V = \pi r^2 h$)

Complete each equation.

10. $(b^2)^{\blacksquare} = b^8$ **11.** $(m^{\blacksquare})^{\frac{1}{3}} = m^{-12}$ **12.** $(x^{\blacksquare})^7 = x^6$

13. $(5x^{\blacksquare})^2 = 25x^{-4}$ **14.** $(3x^3y^{\blacksquare})^3 = 27x^9$ **15.** $(m^2n^3)^{\blacksquare} = \frac{1}{m^6n^9}$

16. How many times the volume of the small cube is the volume of the large cube?



Simplify each expression.

17. $(-5x)^2 + 5x^2$ **18.** $(-2a^{\frac{2}{3}}b)^3(ab^{\frac{1}{3}})^3$

19. $(2x^{-3})^2(0.2x)^2$ **20.** $4xy^2 0^4(-y)^{-3}$

21. $(10^3)^4(4.3 \times 10^{-8})$ **22.** $(3^7)^2(3^{-4})^3$

23. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** Simplify $(x^2)^3$ and x^{2^3}. Are the expressions equivalent? Explain.

Simplify each expression.

24. $(3n^{-6})^{-4}$ **25.** $(7a)^{-2}$ **26.** $(5y^{\frac{1}{2}})^4$ **27.** $(36g^4)^{-\frac{1}{2}}$

28. $(3b^{-2})^2(a^2b^4)^3$ **29.** $4j^2k^6(2j^{11})^3k^5$ **30.** $(mg^4)^{-1}(mg^4)$ **31.** $(2j^2k^4)^{-5}(k^{-1}j^7)^6$

32. a. Explain Mathematical Ideas (1)(G) What mistake did the student make in simplifying the expression at the right?

b. What is the correct simplified form of the expression?

$$(2+3)^2 = 2^2 + 3^2$$
$$= 4 + 9$$
$$= 13$$

STEM **33. Use Representations to Communicate Mathematical Ideas (1)(E)** The power generated by a wind turbine depends on the wind speed. The expression $800v^3$ gives the power in watts for a certain wind turbine at wind speed v in meters per second. If the wind speed triples, by what factor does the power generated by the wind turbine increase?

34. Can you write the expression $49x^2y^2z^2$ using only one exponent? Show how or explain why not.

Simplify. Write each answer in scientific notation.

35. $(7.4 \times 10^4)^2$ **36.** $(6.25 \times 10^{-12})^{-2}$ **37.** $(3.5 \times 10^{-4})^3$

STEM **38. a. Apply Mathematics (1)(A)** Earth has a radius of about 6.4×10^6 m. What is the approximate surface area of Earth? Use the formula for the surface area of a sphere, S.A. $= 4\pi r^2$. Write your answer in scientific notation.

b. Oceans cover about 70% of the surface of Earth. About how many square meters of Earth's surface are covered by ocean water?

c. The oceans have an average depth of 3790 m. Estimate the volume of water in Earth's oceans.

Solve each equation. Use the fact that if $a^x = a^y$, then $x = y$.

39. $5^6 = 25^x$ **40.** $3^x = 27^4$ **41.** $8^{\frac{1}{3}} = 2^x$

42. $4^x = 2^{\frac{1}{2}}$ **43.** $3^{2x} = 9^4$ **44.** $2^x = \frac{1}{32}$

45. Display Mathematical Ideas (1)(G) How many different ways are there to rewrite the expression $16x^4$ using only the property of raising a product to an integer power? Show the ways.

TEXAS End-of-Course PRACTICE

46. Which expression does NOT equal $25n$?

A. $(5n^{\frac{1}{2}})^2$ **B.** $(5n^{\frac{1}{2}})(5n^{\frac{1}{2}})$ **C.** $5(n^2)^{\frac{1}{2}}$ **D.** $5^2(n^{\frac{2}{3}})^{\frac{3}{2}}$

47. A snail travels at a speed of 3×10^{-2} mi/h. What is the snail's speed in inches per minute? Show your work.

5-4 Division Properties of Exponents

TEKS FOCUS

TEKS (11)(B) Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

TEKS (1)(A) **Apply** mathematics to problems arising in everyday life, society, and the workplace.

Additional TEKS (1)(E)

VOCABULARY

- **Apply** – use knowledge or information for a specific purpose, such as solving a problem

ESSENTIAL UNDERSTANDING

You can use properties of exponents to divide powers with the same base.

take note — Property Dividing Powers With the Same Base

Words To divide powers with the same base, subtract the exponents.

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and m and n are rational numbers.

Examples $\frac{2^6}{2^2} = 2^{6-2} = 2^4$ $\qquad \frac{x^4}{x^7} = x^{4-7} = x^{-3} = \frac{1}{x^3}$ $\qquad \frac{s^{\frac{3}{4}}}{s^{\frac{1}{2}}} = s^{\frac{3}{4}-\frac{1}{2}} = s^{\frac{3}{4}-\frac{2}{4}} = s^{\frac{1}{4}}$

take note — Property Raising a Quotient to a Power

Words To raise a quotient to a power, raise the numerator and the denominator to the power and simplify.

Algebra $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $a \neq 0$, $b \neq 0$, and n is a rational number.

Examples $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$ $\qquad \left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$ $\qquad \left(\frac{a}{b}\right)^{\frac{1}{2}} = \frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}$ $\qquad \left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a^n}{b^n}\right)} = \frac{b^n}{a^n}$

Problem 1 TEKS Process Standard (1)(E)

Dividing Algebraic Expressions

What is the simplified form of each expression?

A $\dfrac{x^{\frac{5}{2}}}{x^2}$

$\dfrac{x^{\frac{5}{2}}}{x^2} = x^{\frac{5}{2} - 2}$ Subtract exponents when dividing powers with the same base.

$= x^{\frac{1}{2}}$ Simplify.

B $\dfrac{m^2n^4}{m^5n^3}$

$\dfrac{m^2n^4}{m^5n^3} = m^{2-5}n^{4-3}$ Subtract exponents when dividing powers with the same base.

$= m^{-3}n^1$ Simplify the exponents.

$= \dfrac{n}{m^3}$ Rewrite using positive exponents.

Think

How are the properties for dividing powers and multiplying powers similar?
For both properties, the bases of the powers must be the same. Dividing by a power is the same as multiplying by the power with the opposite exponent.

Problem 2 TEKS Process Standard (1)(A)

Dividing Numbers in Scientific Notation

Demographics **Population density describes the number of people per unit area. During one year, the population of Angola was 1.21×10^7 people. The area of Angola is 4.81×10^5 mi^2. What was the population density of Angola that year?**

Know	Need	Plan
• The population • The area	The population density	Write the ratio of population to area.

$\dfrac{1.21 \times 10^7}{4.81 \times 10^5} = \dfrac{1.21}{4.81} \times 10^{7-5}$ Subtract exponents when dividing powers with the same base.

$= \dfrac{1.21}{4.81} \times 10^2$ Simplify the exponent.

$\approx 0.252 \times 10^2$ Divide. Round to the nearest thousandth.

$= 25.2$ Write in standard notation.

The population density of Angola was about 25.2 people per square mile.

Problem 3

Raising a Quotient to a Power

Multiple Choice What is the simplified form of $\left(\frac{z^{\frac{2}{3}}}{5}\right)^3$?

Ⓐ $\frac{z^{\frac{11}{3}}}{15}$ Ⓑ $\frac{z^2}{15}$ Ⓒ $\frac{z^{\frac{11}{3}}}{125}$ Ⓓ $\frac{z^2}{125}$

$\left(\frac{z^{\frac{2}{3}}}{5}\right)^3 = \frac{\left(z^{\frac{2}{3}}\right)^3}{5^3}$ Raise the numerator and the denominator to the third power.

$= \frac{z^{\frac{2}{3}\cdot 3}}{5^3}$ Multiply the exponents of a power raised to a power.

$= \frac{z^2}{125}$ Simplify.

The correct answer is D.

Think

How can you check your answer?
Substitute the same number for the variable in the original expression and the simplified expression. The expressions should be equal.

Problem 4

Simplifying an Exponential Expression

What is the simplified form of $\left(\frac{2x^6}{y^4}\right)^{-3}$?

$\left(\frac{2x^6}{y^4}\right)^{-3} = \left(\frac{y^4}{2x^6}\right)^3$ Rewrite using the reciprocal of $\frac{2x^6}{y^4}$.

$= \frac{(y^4)^3}{(2x^6)^3}$ Raise the numerator and denominator to the third power.

$= \frac{y^{12}}{8x^{18}}$ Simplify.

Plan

How do you write an expression in simplified form?
Use the properties of exponents to write each variable with a single positive exponent.

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Copy and complete each equation.

1. $\frac{5^9}{5^2} = 5^{■}$

2. $\frac{2^{\frac{7}{3}}}{2^2} = 2^{■}$

3. $\frac{3^2}{3^5} = 3^{■}$

4. $\frac{5^2 \cdot 5^3}{5^3 \cdot 5^2} = 5^{■}$

5. Display Mathematical Arguments (1)(G) The average time it takes a computer to execute one instruction is measured in picoseconds. There are 3.6×10^{15} picoseconds per hour. What fraction of a second is a picosecond? Show your work.

6. Apply Mathematics (1)(A) Data from a deer count in a forested area show that an estimated 3.16×10^3 deer inhabit 7.228×10^4 acres of land. What is the density of the deer population?

STEM **7. Apply Mathematics (1)(A)** The wavelength of a radio wave is defined as speed divided by frequency. An FM radio station has a frequency of 9×10^7 waves per second. The speed of the waves is about 3×10^8 meters per second. What is the wavelength of the station?

8. Justify Mathematical Arguments (1)(G) What mistake did the student make in simplifying the expression at the right? What is the correct simplified form of the expression?

$$5^4 \div 5 = \frac{5^4}{5} = 1^4 = 1$$

9. Explain Mathematical Ideas (1)(G) Suppose $\frac{a^x}{a^y} = a^3$ and $\frac{a^x}{a^{3y}} = a^{-5}$. Find the values of x and y. Explain how you found your answer.

Simplify each expression.

10. $\frac{3^8}{3^6}$ **11.** $\frac{9^{\frac{3}{4}}}{9^{\frac{1}{4}}}$ **12.** $\frac{d^{14}}{d^{17}}$

13. $\frac{3^2m^5t^6}{3^5m^7t^{-5}}$ **14.** $\frac{x^5y^{-\frac{9}{2}}z^3}{xy^{-4}z^3}$ **15.** $\frac{12a^{-1}b^6c^{-3}}{4a^5b^{-1}c^5}$

Simplify each quotient. Write each answer in scientific notation.

16. $\frac{5.2 \times 10^{13}}{1.3 \times 10^7}$ **17.** $\frac{3.6 \times 10^{-10}}{9 \times 10^{-6}}$ **18.** $\frac{6.5 \times 10^4}{5 \times 10^6}$

STEM **19. Apply Mathematics (1)(A)** The sun's mass is 1.998×10^{30} kg. Saturn's mass is 5.69×10^{26} kg. How many times as great as the mass of Saturn is the mass of the sun?

Simplify each expression.

20. $\left(\frac{3}{8}\right)^2$ **21.** $\left(\frac{1}{a}\right)^3$ **22.** $\left(\frac{3x}{y}\right)^4$ **23.** $\left(\frac{2x}{3y}\right)^5$

24. $\left(\frac{2}{5}\right)^{-1}$ **25.** $\left(\frac{5}{4}\right)^{-4}$ **26.** $\left(-\frac{7x^{\frac{3}{2}}}{5y^4}\right)^{-2}$ **27.** $\left(-\frac{2x^{\frac{1}{6}}}{3y^4}\right)^{-3}$

Explain why each expression is *not* in simplest form.

28. 5^3m^3 **29.** x^5y^{-2} **30.** $(2c)^4$ **31.** x^0y **32.** $\frac{d^7}{d}$

33. Apply Mathematics (1)(A) During one year, about 163 million adults over 18 years old in the United States spent a total of about 93 billion hours online at home. On average, how many hours per day did each adult spend online at home?

34. Apply Mathematics (1)(A) During one year, people in the United States older than 18 years old watched a total of 342 billion hours of television. The population of the United States older than 18 years old was about 209 million people.

a. On average, how many hours of television did each person older than 18 years old watch that year? Round to the nearest hour.

b. On average, how many hours per week did each person older than 18 years old watch that year? Round to the nearest hour.

Which property or properties of exponents would you use to simplify each expression?

35. 2^{-3}

36. $\dfrac{2^2}{2^5}$

37. $\dfrac{1}{2^{-4}2^7}$

38. $\dfrac{(2^{\frac{2}{3}})^3}{2^{15}}$

Simplify each expression.

39. $\left(\dfrac{9t^{\frac{2}{3}}}{36t}\right)^3$

40. $\left(\dfrac{a^4a}{a^2}\right)^{-3}$

41. $\left(\dfrac{2x^2}{5x^3}\right)^{-2}$

42. $\dfrac{4x^{-2}y^4}{8x^3(y^{-2})^3}$

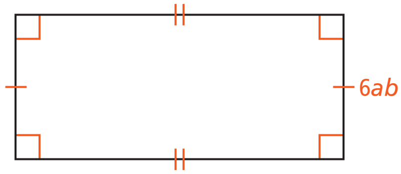
43. a. Display Mathematical Ideas (1)(G) Write three numbers greater than 1000 in scientific notation.

b. Divide each number by 2.

c. Explain Mathematical Ideas (1)(G) Is the exponent of the power of 10 divided by 2 when you divide a number in scientific notation by 2? Explain.

44. Simplify the expression $\left(\dfrac{3}{x^2}\right)^{-3}$ in three different ways. Justify each step.

45. The area of the rectangle is $72a^3b^4$. What is the length of the rectangle?



A. $\dfrac{a^3b^4}{12}$

B. $12a^2b^3$

C. $\dfrac{12}{a^3b^4}$

D. $12a^3b^4$

Simplify each expression.

46. $\dfrac{\left(\frac{1}{4}\right)^{-2}}{\left(\frac{1}{6}\right)^{-3}}$

47. $\dfrac{0.2^3 \cdot 0.2^4}{0.2^7}$

48. $\left(\dfrac{(4x)^2y}{xy^4}\right)^{-2}$

49. $\dfrac{(6a^3)(8b^4)}{(2a^4)(36b^{-1})}$

Write each expression with only one exponent. You may need to use parentheses.

50. $\dfrac{m^7}{n^7}$

51. $\dfrac{10^7 \cdot 10^0}{10^{-3}}$

52. $\dfrac{27x^3}{8y^3}$

53. $\dfrac{4m^2}{169m^4}$

54. **a.** Use the property for dividing powers with the same base to write $\frac{a^0}{a^n}$ as a power of a.

b. Use the definition of a zero exponent to simplify $\frac{a^0}{a^n}$.

c. **Justify Mathematical Arguments (1)(G)** Explain how your results from parts (a) and (b) justify the definition of a negative exponent.

Simplify each expression.

55. $n^{x+2} \div n^x$

56. $n^{5x} \div n^x$

57. $\left(\frac{x^n}{x^{n-2}}\right)^3$

58. $\frac{\left(\frac{m^4}{m^5}\right)}{m^2}$

59. **Explain Mathematical Ideas (1)(G)** Use the division property of exponents to show why 0^0 is undefined.

STEM 60. **Use Representations to Communicate Mathematical Ideas (1)(E)** The density of an object is the ratio of its mass to its volume. Neptune has a mass of 1.02×10^{26} kg. The radius of Neptune is 2.48×10^4 km. What is the density of Neptune in grams per cubic meter? (*Hint:* $V = \frac{4}{3}\pi r^3$)

TEXAS End-of-Course PRACTICE

61. Which expression is equivalent to $\frac{(2x)^5}{x^3}$?

A. $2x^2$

B. $32x^2$

C. $2x^8$

D. $32x^{-2}$

62. Which equation is an equation of the line that contains the point $(8, -3)$ and is perpendicular to the line $y = -4x + 5$?

F. $y = -\frac{1}{4}x - 1$

G. $y = \frac{1}{4}x + \frac{35}{4}$

H. $y = \frac{1}{4}x - 5$

J. $y = 4x - 35$

63. What is the solution of the system of equations $y = -3x + 5$ and $y = -4x - 1$?

A. $(23, 6)$

B. $(6, 23)$

C. $(-6, 23)$

D. $(-6, -23)$

64. You have 8 bags of grass seed. Each bag covers 1200 ft^2 of ground. The function $A(b) = 1200b$ represents the area $A(b)$, in square feet, that b bags cover. What domain and range are reasonable for the function? Explain.

5-5 Rational Exponents and Radicals

TEKS FOCUS

TEKS (11)(B) Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(A)

VOCABULARY

- **Index** – The index is the number n in the crook of the radical sign that gives the degree of the root. If there is no index, the degree is 2, which means square root.
- ***n*th root** – A number b is an nth root of a number a if $b^n = a$.
- **Principal square root** – the nonnegative square root of a number
- **Radical** – an expression made up of a radical sign and a radicand
- **Radical expression** – an expression that contains a radical
- **Radicand** – the number in a radical expression under the radical sign
- **Square root** – A number b is a square root of a number a if $b^2 = a$.

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

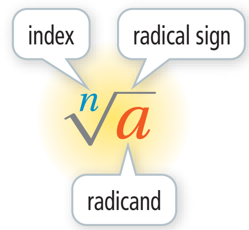
ESSENTIAL UNDERSTANDING

You can use rational exponents to represent radicals.

take note

Key Concept Radicals

A **radical expression** is an expression that contains a radical. The expression under the radical sign is the **radicand**. The number n in the crook of the radical sign is the *index*. The **index** gives the degree of the root. A number b is an ***n*th root** of a number a if $b^n = a$.



If there is no index, the degree is 2, which means square root. A number b is a **square root** of a number a if $b^2 = a$. $\sqrt{a}$ is the nonnegative square root, called the **principal square root**. $-\sqrt{a}$ is the negative square root.

If the nth root of a is a real number and m and n are positive integers, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$.

Problem 1

TEKS Process Standard (1)(E)

Think

How do you find $\sqrt[n]{a}$ for any number a?
You look for n equal factors of a.

Finding Roots

What is the simplified form of each expression?

A $\sqrt[3]{125}$

Method 1

$$\sqrt[3]{125} = \sqrt[3]{5 \cdot 5 \cdot 5}$$
$$= 5$$

B $\sqrt[4]{16}$

Method 2

$$\sqrt[4]{16} = 16^{\frac{1}{4}}$$
$$= (2 \cdot 2 \cdot 2 \cdot 2)^{\frac{1}{4}}$$
$$= 2$$

Problem 2

Plan

Using radical form, do you use the numerator or denominator as the index?
You use the denominator of the rational exponent as the index of the radical.

Converting to Radical Form

A **What is $12a^{\frac{2}{3}}$ in radical form?**

$12a^{\frac{2}{3}} = 12\sqrt[3]{a^2}$ Rewrite $a^{\frac{2}{3}}$ in radical form.

B **What is $(64a)^{\frac{4}{5}}$ in radical form?**

$(64a)^{\frac{4}{5}} = (32 \cdot 2a)^{\frac{4}{5}}$	Since 32 is the 5th power of 2, write $64a$ as a product of 32 and $2a$.
$= 32^{\frac{4}{5}}(2a)^{\frac{4}{5}}$	Power of a product
$= 2^{5 \cdot \frac{4}{5}} \cdot (2a)^{\frac{4}{5}}$	Rewrite 32 as 2^5.
$= 2^4 \cdot (2a)^{\frac{4}{5}}$	Simplify $5 \cdot \frac{4}{5}$.
$= 16\sqrt[5]{(2a)^4}$	Simplify and write $(2a)^{\frac{4}{5}}$ in radical form.
$= 16\sqrt[5]{16a^4}$	Simplify the radicand.

Problem 3

Think

Can you write $\sqrt[3]{27d^5}$ as $27d^{\frac{5}{3}}$?
No. Since the coefficient 27 is under the radical sign, the index operates on the coefficient as well as the variable.

Converting to Exponential Form

A **What is $\sqrt[5]{b^3}$ in exponential form?**

$\sqrt[5]{b^3} = b^{\frac{3}{5}}$ Rewrite using exponential form.

B **What is $\sqrt[3]{27d^5}$ in exponential form? Simplify.**

$\sqrt[3]{27d^5} = (27d^5)^{\frac{1}{3}}$	Rewrite the radical expression in exponential form.
$= 27^{\frac{1}{3}}(d^5)^{\frac{1}{3}}$	Power of a product
$= 3d^{\frac{5}{3}}$	Simplify.

Problem 4

TEKS Process Standard (1)(A)

Using a Radical Expression STEM

Biology **You can estimate the metabolic rate of living organisms based on body mass using Kleiber's law. The formula $R = 73.3\sqrt[4]{M^3}$ relates metabolic rate R measured in Calories per day to body mass M measured in kilograms. What is the metabolic rate of a dog with a body mass of 18 kg?**

Think

How can you find the approximate value of the expression?

You can use $18^{\frac{3}{4}}$ to simplify the radical using a calculator.

$R = 73.3\sqrt[4]{M^3}$

$= 73.3\sqrt[4]{18^3}$ Substitute 18 for M.

≈ 640.5578436 Use a calculator to simplify.

The metabolic rate is about 641 Calories per day.

PRACTICE and APPLICATION EXERCISES

ONLINE HOMEWORK

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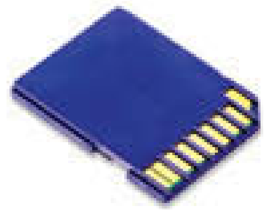
What is the value of each expression?

1. $\sqrt[2]{49}$ **2.** $\sqrt[5]{1}$ **3.** $\sqrt[4]{625}$

4. Apply Mathematics (1)(A) A company that manufactures memory chips for digital cameras uses the formula $c = 120\sqrt[3]{n^2} + 1300$ to determine the cost c, in dollars, of producing n chips. How much will it cost to produce 250 chips?



Write each expression in exponential form.

5. $\sqrt[3]{(8x)^2}$ **6.** $\sqrt[3]{27c^2}$ **7.** $\sqrt[4]{625y^3}$

8. $\sqrt{36x}$ **9.** $\sqrt[4]{x^3}$ **10.** $\sqrt[3]{8b^5}$

STEM **11. Apply Mathematics (1)(A)** Carbon-14 is present in all living organisms and decays at a predictable rate. To estimate the age of an organism, archaeologists measure the amount of carbon-14 left in its remains. The approximate amount of carbon-14 remaining after 5000 years can be found using the formula $A = A_0(2.7)^{-\frac{3}{5}}$, where A_0 is the initial amount of carbon-14 in the sample that is tested. How much carbon-14 is left in a sample that is 5000 years old and originally contained 7.0×10^{-12} grams of carbon-14?

Write each expression in radical form.

12. $a^{\frac{2}{3}}$ **13.** $(64b)^{\frac{3}{4}}$ **14.** $25x^{\frac{1}{2}}$

15. $z^{\frac{3}{4}}$ **16.** $(25x)^{\frac{1}{2}}$ **17.** $27a^{\frac{2}{3}}$

Simplify each expression using the properties of exponents, and then write the expression in radical form.

18. $\left(x^{\frac{3}{4}}\right)\left(x^{\frac{1}{2}}\right)$ **19.** $\left(a^{\frac{2}{3}}\right)\left(a^{\frac{1}{4}}\right)$ **20.** $(cd)^{\frac{1}{2}}\left(d^{\frac{1}{3}}\right)$

21. $(3x^{\frac{1}{3}})(8x^2)$ **22.** $(36x)^{\frac{1}{2}}(49x)^{\frac{1}{2}}$ **23.** $\left(x^{\frac{2}{3}}\right)(8x)^{\frac{1}{3}}$

Write each expression in exponential form. Simplify when possible.

24. $\sqrt[3]{b^2} - \sqrt[3]{b}$ **25.** $3\sqrt[4]{a^3} - 2\sqrt[4]{a^3}$ **26.** $\left(\sqrt[3]{8b^5}\right) - \left(\sqrt[4]{256a^3}\right)$

27. $\sqrt[4]{(9x)^2} + \sqrt[4]{625y^3}$ **28.** $\left(\sqrt[3]{y}\right)\left(\sqrt[3]{y}\right)\left(\sqrt[3]{y}\right)$ **29.** $\sqrt{(2c)^4} + \sqrt[3]{c^6}$

30. Apply Mathematics (1)(A) The radius r of a sphere that has volume V is $r = \sqrt[3]{\frac{3V}{4\pi}}$. The volume of a basketball is approximately 434.67 in.^{3}. The radius of a tennis ball is about one fourth the radius of a basketball. Find the radius of the tennis ball.

31. a. Show that $\sqrt{x^2} = x$ by rewriting $\sqrt{x^2}$ in exponential form.

b. Show that $\sqrt[4]{x^2} = \sqrt{x}$ by rewriting $\sqrt[4]{x^2}$ in exponential form.

32. Explain Mathematical Ideas (1)(G) Explain how to simplify the expression $4x^{\frac{3}{2}} + 3\sqrt{x^3}$. Write the simplified expression in two equivalent forms.

33. Display Mathematical Ideas (1)(G) Write an expression using rational exponents. Then write an equivalent expression using radicals.

34. Apply Mathematics (1)(A) The formula $C = c(1 + r)^n$ can be used to estimate the future cost C of an item due to inflation. Here c represents the current cost of the item, r is the rate of inflation written as a decimal, and n is the number of years for the projection. Suppose a video-game system costs \$299 now. How much will the price increase in nine months with an annual inflation rate of 3.2%?

35. Use Representations to Communicate Mathematical Ideas (1)(E) The number of cells in a cell culture as a function of time is given by the expression $N\left(\frac{6}{5}\right)^t$, where t is measured in hours and N is the initial size of the culture. After 2 hours, there were 144 cells in the culture. What was N? How many cells were in the culture after 20 minutes? How many cells were in the culture after 2.5 hours?

TEXAS End-of-Course PRACTICE

36. Which of the following expressions is equivalent to $(8x)^{\frac{4}{3}}$?

A. $16\sqrt[3]{x^4}$ **C.** $\sqrt[3]{8x^4}$

B. $\sqrt[4]{16x^3}$ **D.** $8\sqrt[4]{x^3}$

37. Which of the following expressions is equivalent to $4\sqrt{b^5}$?

F. $2b^{\frac{2}{5}}$ **H.** $(2b)^{\frac{2}{5}}$

G. $(4b)^{\frac{5}{2}}$ **J.** $4b^{\frac{5}{2}}$

38. Write the expression $\sqrt{9s^3} + \sqrt{16s^3}$ in simplest form with rational exponents.

5-6 Simplifying Radicals

TEKS FOCUS

TEKS (11)(A) Simplify numerical radical expressions involving square roots.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(E)

VOCABULARY

- **Rationalize the denominator** – To rationalize the denominator of an expression, rewrite it so there are no radicals in any denominator and no denominators in any radical.

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

ESSENTIAL UNDERSTANDING

You can simplify radical expressions using multiplication and division properties of square roots.

take note

Property Multiplication Property of Square Roots

Algebra

For $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example

$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

take note

Property Division Property of Square Roots

Algebra

For $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example

$\sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$

take note

Key Concept Simplifying Radicals

A radical expression is simplified if the following statements are true.

- The radicand has no perfect-square factors other than 1.
- The radicand contains no fractions.
- No radicals appear in the denominator of a fraction.

Simplified

$3\sqrt{5}$ $9\sqrt{x}$ $\frac{\sqrt{2}}{4}$

Not Simplified

$3\sqrt{12}$ $\sqrt{\frac{x}{2}}$ $\frac{5}{\sqrt{7}}$

Problem 1

TEKS Process Standard (1)(E)

Removing Perfect-Square Factors

What is the simplified form of $\sqrt{160}$?

$\sqrt{160} = \sqrt{16 \cdot 10}$	16 is the greatest perfect-square factor of 160.
$= \sqrt{16} \cdot \sqrt{10}$	Use the Multiplication Property of Square Roots.
$= 4\sqrt{10}$	Simplify $\sqrt{16}$.

Plan

How can you find the factor to remove?
You can *solve a simpler problem* by listing the factors. Then, choose the greatest perfect square on the list.

Problem 2

TEKS Process Standard (1)(G)

Removing Variable Factors

Multiple Choice **What is the simplified form of $\sqrt{54n^7}$?**

Ⓐ $n^3\sqrt{54n}$ Ⓑ $9n^6\sqrt{6n}$ Ⓒ $3n^3\sqrt{6n}$ Ⓓ $3n\sqrt{27n}$

$\sqrt{54n^7} = \sqrt{9n^6 \cdot 6n}$	$9n^6$, or $(3n^3)^2$, is a perfect-square factor of $54n^7$.
$= \sqrt{9n^6} \cdot \sqrt{6n}$	Use the Multiplication Property of Square Roots.
$= 3n^3\sqrt{6n}$	Simplify $\sqrt{9n^6}$.

The correct answer is C.

Think

How is this problem similar to Problem 1?
In both problems, you need to remove a perfect-square factor from the radicand. In this problem, however, the factor you remove contains a variable.

Problem 3

Multiplying Two Radical Expressions

What is the simplified form of $2\sqrt{7t} \cdot 3\sqrt{14t^2}$?

$2\sqrt{7t} \cdot 3\sqrt{14t^2} = 6\sqrt{7t \cdot 14t^2}$	Multiply the whole numbers and use the Multiplication Property of Square Roots.
$= 6\sqrt{98t^3}$	Simplify under the radical symbol.
$= 6\sqrt{49t^2 \cdot 2t}$	$49t^2$, or $(7t)^2$, is a perfect-square factor of $98t^3$.
$= 6\sqrt{49t^2} \cdot \sqrt{2t}$	Use the Multiplication Property of Square Roots.
$= 6 \cdot 7t\sqrt{2t}$	Simplify $\sqrt{49t^2}$.
$= 42t\sqrt{2t}$	Simplify.

Think

What property allows you to multiply the whole numbers first?
The Commutative Property of Multiplication allows you to change the order of the factors.

Problem 4

Simplifying Fractions Within Radicals

What is the simplified form of each radical expression?

A $\sqrt{\dfrac{64}{49}}$

$\sqrt{\dfrac{64}{49}} = \dfrac{\sqrt{64}}{\sqrt{49}}$	Use the Division Property of Square Roots.
$= \dfrac{8}{7}$	Simplify $\sqrt{64}$ and $\sqrt{49}$.

Think

Which method should you use?
If the denominator is a perfect square, apply the Division Property of Square Roots first. If not, simplify the fraction first.

continued on next page ▶

Problem 4 *continued*

B $\sqrt{\dfrac{8x^3}{50x}}$

$\sqrt{\dfrac{8x^3}{50x}} = \sqrt{\dfrac{4x^2}{25}}$ Divide the numerator and denominator by $2x$.

$= \dfrac{\sqrt{4x^2}}{\sqrt{25}}$ Use the Division Property of Square Roots.

$= \dfrac{\sqrt{4} \cdot \sqrt{x^2}}{\sqrt{25}}$ Use the Multiplication Property of Square Roots.

$= \dfrac{2x}{5}$ Simplify $\sqrt{4}$, $\sqrt{x^2}$, and $\sqrt{25}$.

Problem 5

Rationalizing Denominators

What is the simplified form of each expression?

Think

Does multiplying an expression by $\frac{\sqrt{7}}{\sqrt{7}}$ change its value?
No. The fraction $\frac{\sqrt{7}}{\sqrt{7}}$ is equal to 1. Multiplying an expression by 1 won't change its value.

A $\dfrac{\sqrt{3}}{\sqrt{7}}$

$\dfrac{\sqrt{3}}{\sqrt{7}} = \dfrac{\sqrt{3}}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}}$ Multiply by $\frac{\sqrt{7}}{\sqrt{7}}$.

$= \dfrac{\sqrt{21}}{\sqrt{49}}$

$= \dfrac{\sqrt{21}}{7}$

B $\dfrac{\sqrt{7}}{\sqrt{8n}}$

$\dfrac{\sqrt{7}}{\sqrt{8n}} = \dfrac{\sqrt{7}}{2\sqrt{2n}}$

$= \dfrac{\sqrt{7}}{2\sqrt{2n}} \cdot \dfrac{\sqrt{2n}}{\sqrt{2n}}$ Multiply by $\frac{\sqrt{2n}}{\sqrt{2n}}$.

$= \dfrac{\sqrt{14n}}{2\sqrt{4n^2}}$

$= \dfrac{\sqrt{14n}}{4n}$

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Simplify each product.

1. $\sqrt{8} \cdot \sqrt{32}$ **2.** $-5\sqrt{21} \cdot (-3\sqrt{42})$ **3.** $-\frac{1}{3}\sqrt{18c^5} \cdot \left(-6\sqrt{8c^9}\right)$

4. Suppose a and b are positive integers.

a. Verify that if $a = 18$ and $b = 10$, then $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

b. **Analyze Mathematical Relationships (1)(F)** Find two other pairs of positive integers a and b such that $\sqrt{a} \cdot \sqrt{b} = 6\sqrt{5}$.

Simplify each radical expression.

5. $\dfrac{\sqrt{72}}{\sqrt{64}}$ **6.** $\dfrac{-3\sqrt{2}}{\sqrt{6}}$ **7.** $\sqrt{\dfrac{3m}{16m^2}}$ **8.** $\dfrac{16a}{\sqrt{6a^3}}$

9. **Analyze Mathematical Relationships (1)(F)** What are three numbers whose square roots can be written in the form $a\sqrt{3}$ for some integer value of a?

Simplify each radical expression.

10. $\sqrt{225}$ **11.** $\sqrt{99}$ **12.** $-4\sqrt{117}$ **13.** $3\sqrt{150b^8}$

14. $\sqrt{\frac{16}{25}}$ **15.** $7\sqrt{\frac{6}{32}}$ **16.** $11\sqrt{\frac{49a^5}{4a^3}}$ **17.** $\frac{8\sqrt{7s}}{\sqrt{28s^3}}$

18. **Apply Mathematics (1)(A)** A square picture on the front page of a newspaper occupies an area of 24 in.^{2}. What is the length of each side of the picture? Write your answer as a radical in simplified form.

Explain why each radical expression is or is not in simplified form.

19. $\frac{13x}{\sqrt{4}}$ **20.** $\frac{3}{\sqrt{3}}$ **21.** $-4\sqrt{5}$ **22.** $5\sqrt{30}$

23. **Explain Mathematical Ideas (1)(G)** A student simplified the radical expression at the right. What mistake did the student make? What is the correct answer?

$$\cancel{\sqrt{\frac{5x}{25}} = \frac{\sqrt{5}}{5}x}$$

24. **Display Mathematical Ideas (1)(G)** You can simplify radical expressions with negative exponents by first rewriting the expressions using positive exponents. What are the simplified forms of the following radical expressions?

a. $\frac{\sqrt{3}}{\sqrt{f^{-3}}}$ **b.** $\frac{\sqrt{x^{-3}}}{\sqrt{x}}$ **c.** $\frac{\sqrt{5a^{-2}}}{\sqrt{10a^{-1}}}$ **d.** $\frac{\sqrt{(2m)^{-3}}}{m^{-1}}$

Simplify each radical expression.

25. $\sqrt{24} \cdot \sqrt{2x} \cdot \sqrt{3x}$ **26.** $2b(\sqrt{5b})^2$ **27.** $\sqrt{45a^7} \cdot \sqrt{20a}$

28. **Apply Mathematics (1)(A)** The equation $r = \sqrt{\frac{A}{\pi}}$ gives the radius r of a circle with area A. What is the radius of a circle with the given area? Write your answer as a simplified radical and as a decimal rounded to the nearest hundredth.

a. 50 ft^2 **b.** 32 in.^{2} **c.** 10 m^2

29. **Use Representations to Communicate Mathematical Ideas (1)(E)** For a linear equation in standard form $Ax + By = C$, where $A \neq 0$ and $B \neq 0$, the distance d between the x- and y-intercepts is given by $d = \sqrt{\left(\frac{C}{A}\right)^2 + \left(\frac{C}{B}\right)^2}$. What is the distance between the x- and y-intercepts of the graph of $4x - 3y = 2$?

TEXAS End-of-Course PRACTICE

30. What is the simplified form of $\sqrt{12y^5}$?

A. $2\sqrt{3y^5}$ **B.** $4y^4\sqrt{3y}$ **C.** $2y^2\sqrt{3y}$ **D.** $3y^2$

31. In the proportion $\frac{3}{b} = \frac{7}{8 - b}$, what is the value of b?

F. 6 **G.** $\frac{21}{8}$ **H.** $\frac{12}{5}$ **J.** $\frac{5}{12}$

5-7 The Pythagorean Theorem

TEKS FOCUS

TEKS (11)(A) Simplify numerical radical expressions involving square roots.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(A), (8)(A)

VOCABULARY

- **Hypotenuse** – The hypotenuse is the side opposite the right angle in a right triangle. It is the longest side in the triangle.
- **Leg** – each of the sides that form the right angle of a right triangle
- **Pythagorean Theorem** – In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse: $a^2 + b^2 = c^2$.

- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

The lengths of the sides of a right triangle have a special relationship. If you know the lengths of any two of the sides, you can find the length of the third side.

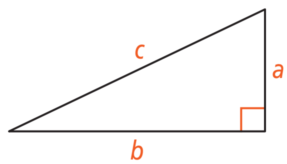
take note

Key Concept The Pythagorean Theorem

Words
In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Algebra
$a^2 + b^2 = c^2$

Diagram



take note

Key Concept The Converse of the Pythagorean Theorem

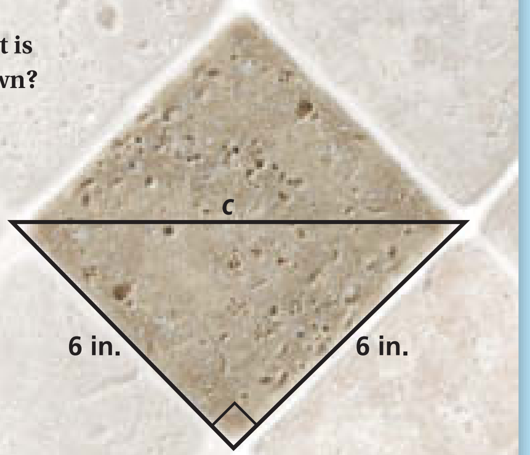
If a triangle has sides of lengths a, b, and c, and $a^2 + b^2 = c^2$, then the triangle is a right triangle with hypotenuse of length c.

Problem 1

TEKS Process Standard (1)(A)

Finding the Length of a Hypotenuse

The tiles at the right are squares with 6-in. sides. What is the length of the hypotenuse of the right triangle shown? Give your answer in simplified radical form and as a decimal rounded to the nearest tenth.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + 6^2 = c^2$	Substitute 6 for *a* and *b*.
$72 = c^2$	Simplify.
$\sqrt{72} = \sqrt{c^2}$	Find the principal square root.
$6\sqrt{2} = c$	Find the simplified radical form.
$8.5 \approx c$	Use a calculator.

The length of the hypotenuse is $6\sqrt{2}$ in., or about 8.5 in.

Plan

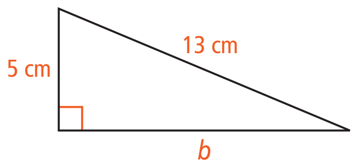
What do you know? What do you need?
You know the lengths *a* and *b* of the two legs. You need to find the length *c* of the hypotenuse. Substitute for *a* and *b* in $a^2 + b^2 = c^2$, and then solve for *c*.

Problem 2

TEKS Process Standard (1)(D)

Finding the Length of a Leg

What is the side length *b* in the triangle below?



$a^2 + b^2 = c^2$	Pythagorean Theorem
$5^2 + b^2 = 13^2$	Substitute 5 for *a* and 13 for *c*.
$25 + b^2 = 169$	Simplify.
$b^2 = 144$	Subtract 25 from each side.
$\sqrt{b^2} = \sqrt{144}$	Find the principal square root of each side.
$b = 12$	Simplify.

The side length *b* is 12 cm.

Think

How is this problem different from Problem 1?
In Problem 1, the length of the hypotenuse was unknown. In this problem, the length of a leg is unknown.

Problem 3

Identifying Right Triangles

Plan

Why should you check each answer choice?
If you find two answer choices that appear to be correct, then you know you have made a mistake.

Multiple Choice **Which set of lengths could be the side lengths of a right triangle?**

Ⓐ 6 in., 24 in., $18\sqrt{2}$ in.

Ⓑ $5\sqrt{3}$ m, 8 m, 10 m

Ⓒ 4 in., $4\sqrt{3}$ in., 8 in.

Ⓓ 8 ft, 16 ft, $12\sqrt{2}$ ft

Determine whether the lengths satisfy $a^2 + b^2 = c^2$. The greatest length is c.

$$6^2 + 24^2 \stackrel{?}{=} (18\sqrt{2})^2 \qquad (5\sqrt{3})^2 + 8^2 \stackrel{?}{=} 10^2$$
$$36 + 576 \stackrel{?}{=} 324 \cdot 2 \qquad 25 \cdot 3 + 64 \stackrel{?}{=} 100$$
$$612 \neq 648 \qquad 139 \neq 100$$

$$4^2 + (4\sqrt{3})^2 \stackrel{?}{=} 8^2 \qquad 8^2 + 16^2 \stackrel{?}{=} (12\sqrt{2})^2$$
$$16 + 16 \cdot 3 \stackrel{?}{=} 64 \qquad 64 + 256 \stackrel{?}{=} 144 \cdot 2$$
$$64 = 64 \ \checkmark \qquad 320 \neq 288$$

By the Converse of the Pythagorean Theorem, the lengths 4 in., $4\sqrt{3}$ in., and 8 in. could be the side lengths of a right triangle. The correct answer is C.

Problem 4

Writing a Radical Expression

Art **A rectangular door in a museum is three times as tall as it is wide. What is a simplified expression for the maximum length of a painting that fits through the door?**

Know
The door is w units wide and $3w$ units high.

Need
The diagonal length d of the doorway

Plan
Use the Pythagorean Theorem.

Think

How is this like problems you have done before?
The width and height of the door are two legs of a right triangle. You can find the hypotenuse of a right triangle using the Pythagorean Theorem.

$d^2 = w^2 + (3w)^2$	Pythagorean Theorem
$d^2 = w^2 + 9w^2$	Simplify $(3w)^2$.
$d^2 = 10w^2$	Combine like terms.
$d = \sqrt{10w^2}$	Find the principal square root of each side.
$d = \sqrt{w^2} \cdot \sqrt{10}$	Multiplication Property of Square Roots
$d = w\sqrt{10}$	Simplify $\sqrt{w^2}$.

An expression for the maximum length of the painting is $w\sqrt{10}$, or about $3.16w$.

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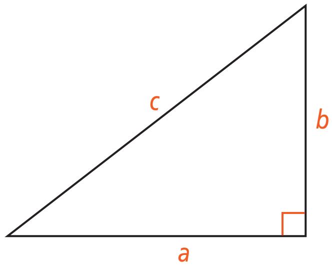
Use the triangle at the right. Find the missing side length. Give your answer in simplified radical form and as a decimal rounded to the nearest tenth.



1. $a = 3, b = 4$ **2.** $a = 6, c = 10$ **3.** $b = 1, c = \frac{5}{4}$

4. $a = 5, c = 13$ **5.** $a = 0.3, b = 0.4$ **6.** $a = 3, b = 6$

7. $a = 7, b = 7$ **8.** $b = 8, c = 12$ **9.** $b = 3.5, c = 3.7$

10. Apply Mathematics (1)(A) A jogger goes half a mile north and then turns west. If the jogger finishes 1.3 mi from the starting point, how far west did the jogger go?

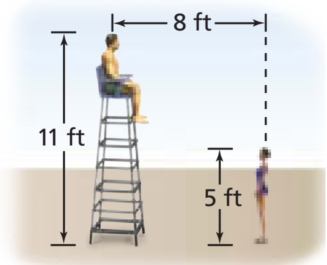
STEM **11. Apply Mathematics (1)(A)** A construction worker is cutting along the diagonal of a rectangular board 15 ft long and 8 ft wide. What will be the length of the cut?

Determine whether the given lengths can be side lengths of a right triangle.

12. $5\sqrt{5}$ ft, 10 ft, 15 ft **13.** $7\sqrt{2}$ m, 12 m, 16 m **14.** 13 in., 35 in., 38 in.

15. 16 cm, 63 cm, 65 cm **16.** 6 in., 12 in., $6\sqrt{5}$ in. **17.** 16 yd, 30 yd, 34 yd

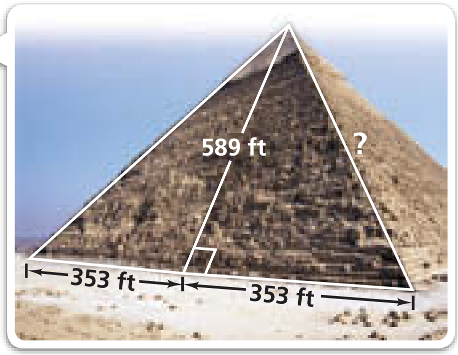
18. Apply Mathematics (1)(A) A swimmer asks a question to a lifeguard sitting on a tall chair, as shown in the diagram at the right. The swimmer needs to be close to the lifeguard to hear the answer. What is the distance between the swimmer's head and the lifeguard's head?



STEM **19. Use a Problem-Solving Model (1)(B)** Students are building rectangular wooden frames for the set of a school play. The height of a frame is 6 times the width w. Each frame has a brace that connects two opposite corners of the frame. What is a simplified expression for the length of a brace?

20. Apply Mathematics (1)(A) A park is shaped like a rectangle with a length 5 times its width w. What is a simplified expression for the distance between opposite corners of the park?

21. Apply Mathematics (1)(A) Originally, each face of the Pyramid of Khafre was a triangle with the dimensions shown. How far was a corner of the base from the pyramid's top? Round to the nearest foot.



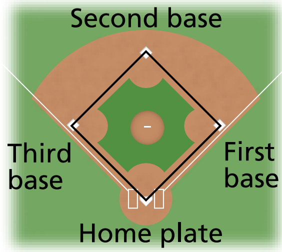
Any set of three positive integers that satisfies the equation $a^2 + b^2 = c^2$ is a *Pythagorean triple*. Determine whether each set of numbers is a Pythagorean triple.

22. 11, 60, 61 **23.** 13, 84, 85

24. 40, 41, 58 **25.** 50, 120, 130

26. 32, 126, 130 **27.** 28, 45, 53

28. **Apply Mathematics (1)(A)** The bases in a softball diamond are located at the corners of a 3600-ft^2 square. How far is a throw from second base to home plate?



29. **Use a Problem-Solving Model (1)(B)** A banner shaped like a right triangle has a hypotenuse of length 26 ft and a leg of length 10 ft. What is the area of the banner?

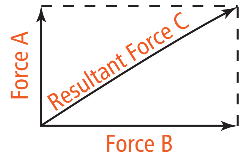
30. Two sides of a right triangle measure 10 in. and 8 in.

    a. **Explain Mathematical Ideas (1)(G)** Explain why this is not enough information to be sure of the length of the third side.

    b. Give two possible values for the length of the third side.

31. A rectangular box is 4 cm wide, 4 cm tall, and 10 cm long. What is the diameter of the smallest circular opening through which the box will fit? Round to the nearest tenth of a centimeter.

STEM 32. **Apply Mathematics (1)(A)** If two forces pull at right angles to each other, the resultant force can be represented by the diagonal of a rectangle, as shown at the right. This diagonal is a hypotenuse of a right triangle. A 50-lb force and a 120-lb force combine for a resultant force of 130 lb. Are the forces pulling at right angles to each other? Explain.



33. **Display Mathematical Ideas (1)(G)** From a viewing height of h feet, the approximate distance d to the horizon, in miles, is given by the equation $d = \sqrt{\frac{3h}{2}}$. To the nearest mile, what is the distance to the horizon from a height of 150 ft? 225 ft? 300 ft?

TEXAS End-of-Course PRACTICE

34. A park has two walking paths shaped like right triangles. The first path has legs 75 yd and 100 yd long. The second path has legs 50 yd and 240 yd long. What is the total length of the shorter path, in yards?

35. What is the slope of the graph of the equation $y = \frac{1}{2}x + 7$?

36. A candidate in an election received 72.5% of the vote. What decimal represents the portion of the voters who did NOT vote for the candidate?

Topic 5 Review

TOPIC VOCABULARY

- hypotenuse, *p. 233*
- index, *p. 225*
- leg, *p. 233*
- *n*th root, *p. 225*
- principal square root, *p. 225*
- Pythagorean Theorem, *p. 233*
- radical, *p. 225*
- radical expression, *p. 225*
- radicand, *p. 225*
- rationalize the denominator, *p. 229*
- square root, *p. 225*

Check Your Understanding

1. A(n) __?__ is the number in the crook of the radical sign that gives the degree of the root.

2. To __?__, rewrite the expression so there are no radicals in any denominator and no denominators in any radical.

3. A(n) __?__ is an expression that contains a radical.

4. The side opposite the right angle in a right triangle is the __?__.

5. The __?__ states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

5-1 Zero and Negative Exponents

Quick Review

You can use zero and negative integers as exponents. For every nonzero number a, $a^0 = 1$. For every nonzero number a and any rational number n, $a^{-n} = \frac{1}{a^n}$. When you evaluate an exponential expression, you can simplify the expression before substituting values for the variables.

Example

What is the value of $a^2b^{-4}c^0$ for $a = 3$, $b = 2$, and $c = -5$?

$a^2b^{-4}c^0 = \frac{a^2c^0}{b^4}$ Use the definition of negative exponents.

$= \frac{a^2(1)}{b^4}$ Use the definition of zero exponent.

$= \frac{3^2}{2^4}$ Substitute.

$= \frac{9}{16}$ Simplify.

Exercises

Simplify each expression.

6. 5^0

7. 7^{-2}

8. $\frac{4x^{-2}}{y^{-8}}$

9. $\frac{1}{p^2q^{-4}r^0}$

Evaluate each expression for $x = 2$, $y = -3$, and $z = -5$.

10. x^0y^2

11. $(-x)^{-4}y^2$

12. x^0z^0

13. $\frac{5x^0}{y^{-2}}$

14. $y^{-2}z^2$

15. $\frac{2x}{y^2z^{-1}}$

16. Is it true that $(-3b)^4 = -12b^4$? Explain why or why not.

5-2 Multiplying Powers With the Same Base

Quick Review

To multiply powers with the same base, add the exponents.

$a^m \cdot a^n = a^{m+n}$, where $a \neq 0$ and m and n are rational numbers.

Example

What is the simplified form of each expression?

a. $3^{10} \cdot 3^4 = 3^{10+4} = 3^{14}$

b. $\left(a^4\right)\left(a^3\right) = a^{4+3} = a^7$

c. $\left(x^{\frac{3}{5}}\right)\left(x^{\frac{1}{5}}\right) = x^{\frac{3}{5}+\frac{1}{5}} = x^{\frac{4}{5}}$

d. $\left(b^{\frac{3}{4}}\right)\left(b^{\frac{1}{2}}\right) = b^{\frac{3}{4}+\frac{1}{2}} = b^{\frac{3}{4}+\frac{2}{4}} = b^{\frac{5}{4}}$

Exercises

Complete each equation.

17. $3^2 \cdot 3^{\blacksquare} = 3^{10}$

18. $a^6 \cdot a^{\blacksquare} = a^8$

19. $x^2y^5 \cdot x^{\blacksquare}y^{\blacksquare} = x^5y^{11}$

20. $a^{\frac{1}{2}} \cdot a^{\blacksquare} = a$

21. $x^{\frac{2}{3}} \cdot x^{\blacksquare} = x^{\frac{11}{12}}$

22. $m^{\frac{3}{4}}n^{\frac{1}{2}} \cdot m^{\blacksquare}n^{\blacksquare} = m^{\frac{5}{4}}n$

Simplify each expression.

23. $2d^2 \cdot d^3$

24. $(x^3)(x^4)$

25. $\left(x^3y^5\right)\left(-y^7x\right)$

26. $\left(s^{\frac{3}{5}}\right)\left(s^{\frac{2}{3}}\right)$

27. $\left(p^{\frac{1}{3}}q\right)\left(q^{\frac{1}{2}}p\right)$

28. $2m^{\frac{3}{4}}n^2 \cdot 3m^{\frac{1}{4}}n$

29. Each square inch of your body has about 6.5×10^2 pores. Suppose the back of your hand has an area of about 0.12×10^2 in.^{2}. About how many pores are on the back of your hand? Write your answer in scientific notation.

5-3 More Multiplication Properties of Exponents

Quick Review

To raise a power to a power, multiply the exponents.

$(a^m)^n = a^{mn}$, where $a \neq 0$ and m and n are rational numbers.

To raise a product to a power, raise each factor in the product to the power.

$(ab)^n = a^nb^n$, where $a \neq 0$, $b \neq 0$, and n is a rational number.

Example

What is the simplified form of each expression?

a. $(x^5)^7 = x^{5 \cdot 7} = x^{35}$

b. $(pq)^8 = p^8q^8$

c. $\left(x^{\frac{1}{3}}\right)^3 = x^{\frac{1}{3} \cdot 3} = x^{\frac{3}{3}} = x$

d. $\left(ab\right)^{\frac{2}{3}} = a^{\frac{2}{3}}b^{\frac{2}{3}}$

Exercises

Complete each equation.

30. $(5^5)^{\blacksquare} = 5^{15}$

31. $(b^{-4})^{\blacksquare} = b^{20}$

32. $(4x^3y^5)^{\blacksquare} = 16x^6y^{10}$

33. $\left(x^{\frac{2}{3}}\right)^{\blacksquare} = x^2$

34. $\left(a^{\frac{1}{2}}\right)^{\blacksquare} = a^{\frac{1}{4}}$

35. $\left(2x^2y^{\frac{1}{4}}\right)^{\blacksquare} = 4x^4y^{\frac{1}{2}}$

Simplify each expression.

36. $(q^3r)^4$

37. $(1.34^2)^5(1.34)^{-8}$

38. $(12x^2y^{-2})^5(4xy^{-3})^{-7}$

39. $(-2r^{-4})^2(-3r^2z^8)^{-1}$

40. $\left(x^{\frac{4}{7}}\right)^7$

41. $\left(a^{\frac{3}{4}}b^{\frac{7}{8}}\right)^4$

5-4 Division Properties of Exponents

Quick Review

To divide powers with the same base, subtract the exponents.

$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and m and n are rational numbers.

To raise a quotient to a power, raise the numerator and the denominator to the power.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $a \neq 0$, $b \neq 0$, and n is a rational number.

Example

What is the simplified form of $\left(\frac{5x^4}{z^2}\right)^3$?

$$\left(\frac{5x^4}{z^2}\right)^3 = \frac{(5x^4)^3}{(z^2)^3} = \frac{5^3x^{4\cdot 3}}{z^{2\cdot 3}} = \frac{125x^{12}}{z^6}$$

Exercises

Simplify each expression.

42. $\frac{w^2}{w^5}$

43. $\frac{21x^3}{3x^{-1}}$

44. $\left(\frac{n^5}{v^3}\right)^7$

45. $\left(\frac{3c^3}{e^5}\right)^{-4}$

Simplify each quotient. Write your answer in scientific notation.

46. $\frac{4.2 \times 10^8}{2.1 \times 10^{11}}$

47. $\frac{3.1 \times 10^4}{1.24 \times 10^2}$

48. $\frac{4.5 \times 10^3}{9 \times 10^7}$

49. $\frac{5.1 \times 10^5}{1.7 \times 10^2}$

50. List the steps that you would use to simplify $\left(\frac{5a^8}{10a^6}\right)^{-3}$.

5-5 Rational Exponents and Radicals

Quick Review

If the nth root of a is a real number and m and n are positive integers, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

Example

Write the expression $(8x)^{\frac{1}{3}}$ in radical form.

$(8x)^{\frac{1}{3}} = 8^{\frac{1}{3}}x^{\frac{1}{3}} = 2\sqrt[3]{x}$

Write the expression $\sqrt[3]{b^2}$ as a power with a rational exponent.

$\sqrt[3]{b^2} = b^{\frac{2}{3}}$

Exercises

Write each expression in radical form.

51. $m^{\frac{1}{2}}$

52. $p^{\frac{2}{3}}r^{\frac{4}{5}}$

53. $\left(36x^4\right)^{\frac{1}{2}}$

54. $(125x)^{\frac{1}{3}}$

55. $(64)^{\frac{1}{2}}x^{\frac{3}{4}}$

56. $25^{\frac{1}{3}}\left(x^2y\right)^{\frac{1}{2}}$

Write each expression as a power with a rational exponent.

57. $\sqrt{xy}$

58. $\sqrt[4]{a}$

59. $\sqrt[3]{b^2}$

60. $\sqrt[3]{x^6y^9}$

61. $\sqrt[4]{81x^2}$

62. $\sqrt[5]{x^2y^3}$

5-6 Simplifying Radicals

Quick Review

A **radical expression** is simplified if the following statements are true.

- The radicand has no perfect-square factors other than 1.
- The radicand contains no fractions.
- No radicals appear in the denominator of a fraction.

Example

What is the simplified form of $\frac{\sqrt{3x}}{\sqrt{2}}$?

$$\frac{\sqrt{3x}}{\sqrt{2}} = \frac{\sqrt{3x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.$$

$$= \frac{\sqrt{6x}}{\sqrt{4}} \quad \text{Multiply numerators and denominators.}$$

$$= \frac{\sqrt{6x}}{2} \quad \text{Simplify.}$$

Exercises

Simplify each radical expression.

63. $3\sqrt{14} \cdot (-2\sqrt{21})$

64. $\sqrt{8} \cdot \frac{1}{4}\sqrt{6}$

65. $\sqrt{\frac{25a^3}{4a}}$

66. $\frac{\sqrt{8s}}{\sqrt{18s^3}}$

67. $-2\sqrt{7x^2} \cdot \frac{1}{3}\sqrt{28x^3}$

68. $6\sqrt{5t^3} \cdot \sqrt{15t^5}$

69. Write three radical expressions that have $4\sqrt{2s}$ as their simplified form. What do the three expressions have in common? Explain.

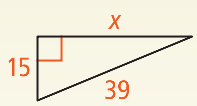
5-7 The Pythagorean Theorem

Quick Review

Given the lengths of two sides of a right triangle, you can use the **Pythagorean Theorem** to find the length of the third side. Given the lengths of all three sides of a triangle, you can determine whether it is a right triangle.

Example

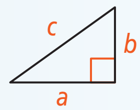
What is the side length x in the triangle at the right?



$a^2 + b^2 = c^2$	Pythagorean Theorem
$15^2 + x^2 = 39^2$	Substitute 15 for *a*, *x* for *b*, and 39 for *c*.
$225 + x^2 = 1521$	Simplify.
$x^2 = 1296$	Subtract 225 from each side.
$x = 36$	Find the principal square root of each side.

Exercises

Use the triangle at the right. Find the missing side length. Give your answer in simplified radical form and as a decimal rounded to the nearest tenth.



70. $a = 12, b = 6$

71. $a = 3\frac{1}{2}, b = 12$

72. $a = 1.1, b = 6$

73. $a = 13, c = 85$

74. $a = 6, c = 3\sqrt{5}$

75. $b = 14, c = 21$

76. $b = 8.8, c = 11$

77. $a = 3, c = 3\sqrt{2}$

Determine whether the given lengths can be side lengths of a right triangle.

78. $4, 2\sqrt{5}, 6$

79. 22, 120, 122

80. 8, 40, 41

81. $1\frac{2}{5}, 3, 3\frac{2}{5}$

82. 6, 24, 25

83. $18, 36, 18\sqrt{5}$

84. 1.2, 6, 6.1

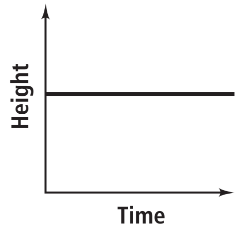
85. $4, 7\sqrt{3}, 12$

86. 1.3, 8.4, 8.5

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. The graph at the right shows how Manuel's height changed during the past year. Which conclusion can you make from the graph?



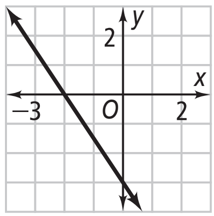
A. His height is average.

B. He will grow more next year.

C. His height did not change during the year.

D. His height steadily increased during the year.

2. What is the y-intercept of the graph at the right?



F. -3 **H.** $-\frac{3}{2}$

G. -2 **J.** 0

3. Which *cannot* be represented by a linear function?

A. the area of a square, given its side length

B. the price of fruit, given the weight of the fruit

C. the number of steps on a ladder, given the height

D. the number of inches, given the number of yards

4. A light-year is the distance light travels in one year. One light-year is about 5.9×10^{12} mi. If it takes light 3 months to travel from one star to another, about how far apart are the stars?

F. 2×10^3 mi **H.** 1.5×10^{12} mi

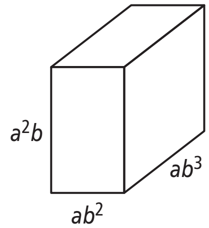
G. 1.5×10^4 mi **J.** 2×10^{12} mi

5. At lunchtime, Mitchell cast a shadow 0.5 ft long while a nearby flagpole cast a shadow 2.5 ft long. If Mitchell is 5 ft 3 in. tall, how tall is the flagpole?

A. 26 ft 3 in. **C.** 26 ft 5 in.

B. 26 ft 4 in. **D.** 26 ft 6 in.

6. The dimensions of a rectangular prism are shown in the diagram at the right. Which expression represents the volume of the rectangular prism?



F. a^2b^5 **H.** a^4b^5

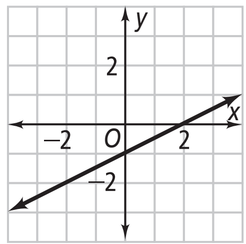
G. a^2b^6 **J.** a^4b^6

7. Suppose you are buying apples and bananas. The price of apples is \$.40 each and the price of bananas is \$.25 each. Which equation models the number of apples and bananas you can buy for \$2?

A. $40x + 25y = 200$ **C.** $5x + 8y = 200$

B. $40x - 25y = 2$ **D.** $5x + 8y = 2$

8. Use the graph at the right. Suppose the y-intercept decreases by 3 and the slope stays the same. What will the x-intercept be?



F. -6 **H.** 2

G. -3 **J.** 3

9. Laura rented a car that cost \$20 for the day plus \$.12 for each mile driven. She returned the car later that day. Laura gave the salesperson \$50 and received change. Which inequality represents the possible numbers of miles m that she could have driven?

A. $50 > 0.12m + 20$ **C.** $50 > 0.12m - 20$

B. $50 < 0.12m + 20$ **D.** $50 < 0.12m - 20$

10. A doctor did a 6-month study on resting heart rate and exercise in healthy adults. The doctor found that for every 20 min of exercise added to a daily routine, the resting heart rate decreased by 1 beat per minute. According to the doctor's study, what does the resting heart rate depend on?

F. the 6-month study **H.** a daily routine

G. minutes of exercise **J.** diet

11. Which expression is equivalent to $\sqrt[4]{81x^3}$?

A. $81x^{\frac{3}{4}}$

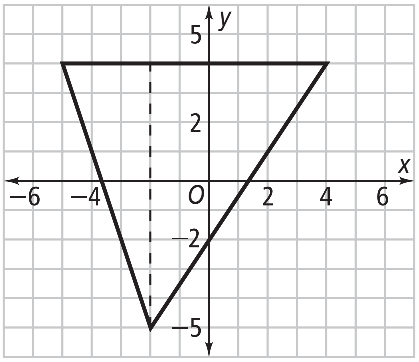
B. $3x^{\frac{3}{4}}$

C. $3x^3$

D. $(3x)^{\frac{3}{4}}$

Gridded Response

12. What is the area, in square units, of the triangle below?



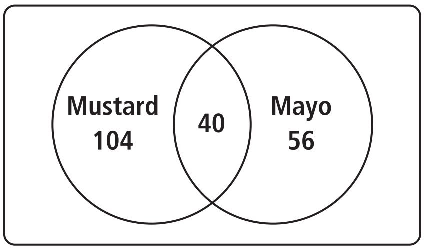
13. Charles purchased 50 shares of a stock at \$23 per share. He paid a \$15 commission to his broker for the purchase. How much money, in dollars, did he spend for the purchase and commission combined?

14. What is the value of the expression $\left(8^{\frac{1}{3}}\right)^2$?

15. A parallelogram has vertices $(-3, 2)$, $(0, 7)$, $(7, 7)$, and $(x, 2)$. What is the value of x if $x > 0$?

16. Alejandro bought 6 notebooks and 2 binders for \$23.52. Cassie bought 3 notebooks and 4 binders for \$25.53. What was the cost, in dollars, of 1 notebook?

17. Ashley surveyed 200 students in her school to find out whether they liked mustard or mayo on a turkey sandwich. Her results are shown in the diagram below.



What percent of the students surveyed liked mustard but not mayo?

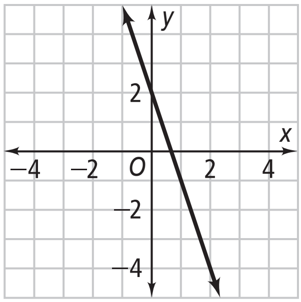
Constructed Response

18. If $b = 2a - 16$ and $b = a + 2$, what is $a + b$?

19. The sum of six consecutive integers is 165. What are the six integers? Show your work.

20. On April 1, 2000, the day of the 2000 national census, the population of the United States was 281,421,906 people. This was a 13.2% increase from the 1990 census. What was the 1990 population of the United States?

21. What is the equation of the graph in standard form?



22. A triangle is enclosed by the following lines:

$$x - y = -1$$
$$y = 2$$
$$-0.4x - y = -5.2$$

a. What are the coordinates of the vertices of the triangle? Use algebraic methods to justify your answers.

b. Draw the triangle using your answers in part (a). What is the area, in square units, of the triangle?

23. On Kids' Night, there must be at least one child with every adult. The restaurant has a maximum seating capacity of 75 people.

a. Write a system of inequalities to represent this situation.

b. Graph the solution. Is it possible for 50 children to accompany 10 adults to the restaurant?

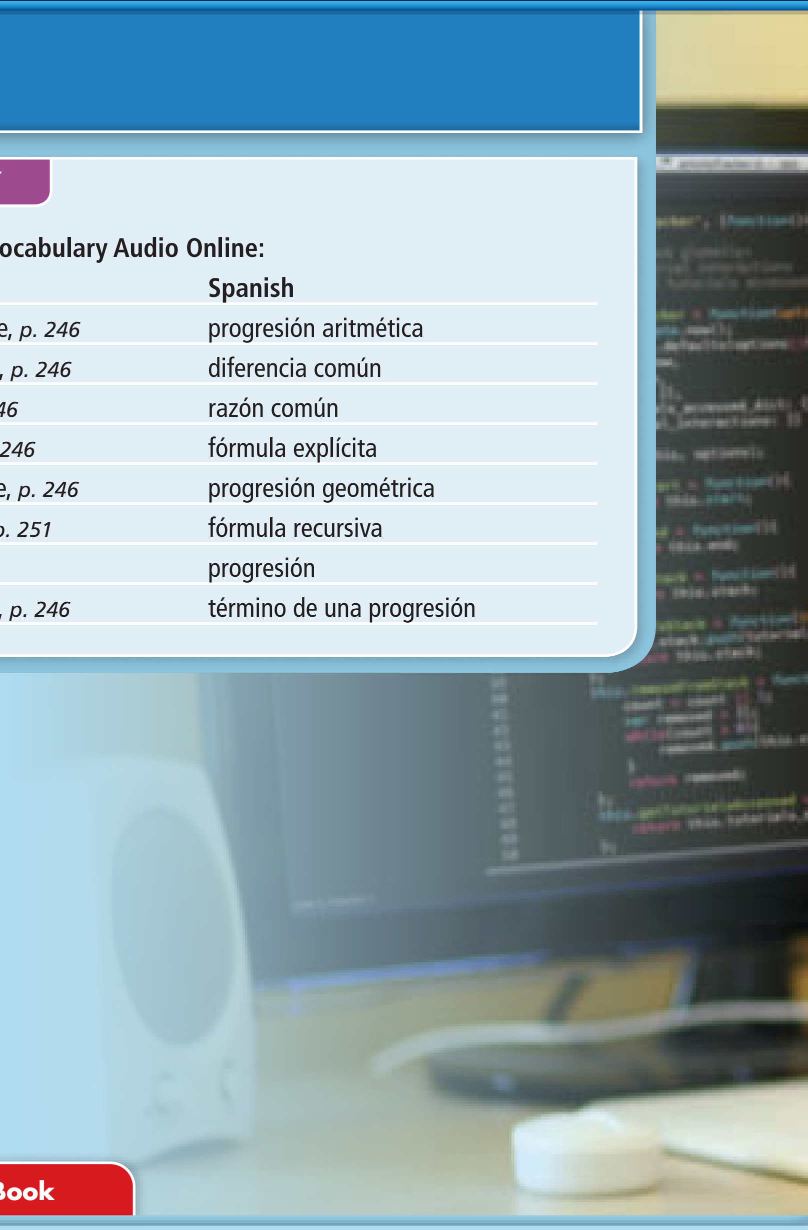
Topic 6 Sequences

TOPIC OVERVIEW

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
arithmetic sequence, *p. 246*	progresión aritmética
common difference, *p. 246*	diferencia común
common ratio, *p. 246*	razón común
explicit formula, *p. 246*	fórmula explícita
geometric sequence, *p. 246*	progresión geométrica
recursive formula, *p. 251*	fórmula recursiva
sequence, *p. 246*	progresión
term of a sequence, *p. 246*	término de una progresión



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STUDENT TEXT AND HOMEWORK HELPER

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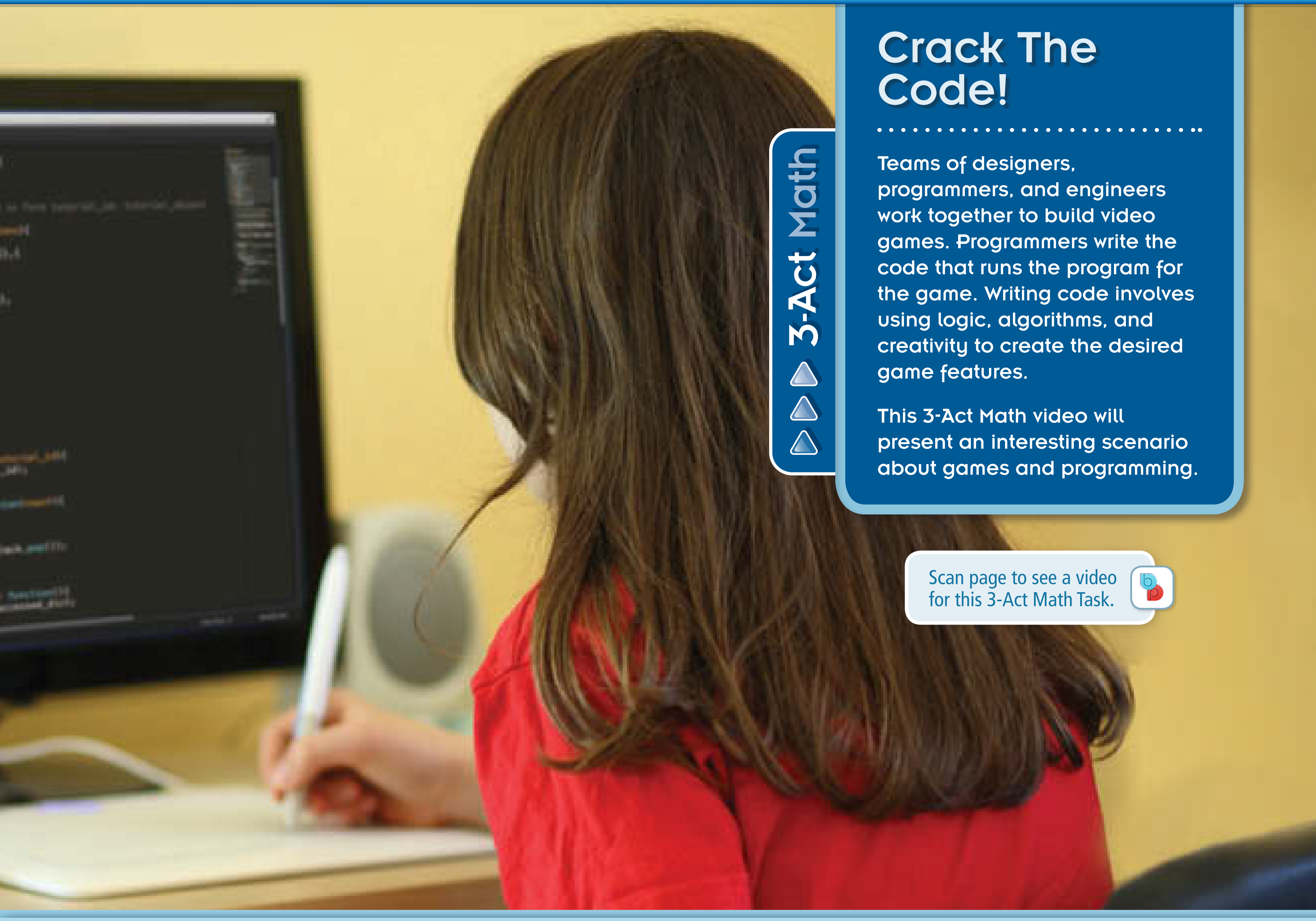

3-Act Math

Crack The Code!

Teams of designers, programmers, and engineers work together to build video games. Programmers write the code that runs the program for the game. Writing code involves using logic, algorithms, and creativity to create the desired game features.

This 3-Act Math video will present an interesting scenario about games and programming.

Scan page to see a video for this 3-Act Math Task.

If You Need Help...

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You'll find definitions of math terms in both English and Spanish. All of the terms have audio support.

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Not sure how to do some of the practice exercises? Check out the Virtual Nerd videos for stepped-out, multi-level instructional support.

6-1 Arithmetic and Geometric Sequences

TEKS FOCUS

TEKS (12)(D) Write a formula for the *n*th term of arithmetic and geometric sequences, given the value of several of their terms.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(A), (1)(F), (12)(B)

VOCABULARY

- **Arithmetic sequence** – a number sequence formed by adding a fixed number to each term of the sequence to find the next term
- **Common difference** – the difference between consecutive terms of an arithmetic sequence
- **Common ratio** – the ratio between consecutive terms of a geometric sequence
- **Explicit formula** – An explicit formula expresses the *n*th term of a sequence in terms of *n*.
- **Geometric sequence** – a number sequence formed by multiplying each term in the sequence by a fixed number to find the next term
- **Sequence** – an ordered list of numbers that often form a pattern
- **Term of a sequence** – A term of a sequence is any number in a sequence.

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

When you can identify a pattern in a sequence, you can use it to extend the sequence. You can also model some sequences with a function rule that you can use to find any term of the sequence.

take note

Key Concept Explicit Formula for an Arithmetic Sequence

The *n*th term of an arithmetic sequence with first term $A(1)$ and common difference d is given by

$$\underbrace{A(n)}_{n\text{th term}} = \underbrace{A(1)}_{\text{first term}} + (\underbrace{n}_{\text{term number}} - 1)\underbrace{d}_{\text{common difference}}$$

take note

Key Concept Explicit Formula for a Geometric Sequence

The *n*th term of a geometric sequence with first term $A(1)$ and common ratio r is given by

$$A(n) = A(1) \cdot r^{n-1}, \text{ for } n \geq 1$$

Problem 1

Plan

How can you identify a pattern?
Look at how each term of the sequence is related to the previous term. Your goal is to identify a single rule that you can apply to every term to produce the next term.

Extending Sequences

Describe a pattern in each sequence. What are the next two terms of each sequence?

A $5, \xrightarrow{+3} 8, \xrightarrow{+3} 11, \xrightarrow{+3} 14, \dots$

A pattern is "add 3 to the previous term." So the next two terms are $14 + 3 = 17$ and $17 + 3 = 20$.

B $2.5, \xrightarrow{\times 2} 5, \xrightarrow{\times 2} 10, \xrightarrow{\times 2} 20, \dots$

A pattern is "multiply the previous term by 2." So the next two terms are $2(20) = 40$ and $2(40) = 80$.

Problem 2

TEKS Process Standard (1)(F)

Plan

How can you identify an arithmetic sequence?
The difference between every pair of consecutive terms must be the same.

Identifying an Arithmetic Sequence

Tell whether the sequence is arithmetic. If it is, what is the common difference?

A $3, \xrightarrow{+5} 8, \xrightarrow{+5} 13, \xrightarrow{+5} 18, \dots$

The sequence has a common difference, $d = 5$. So it is arithmetic.

B $6, \xrightarrow{+3} 9, \xrightarrow{+4} 13, \xrightarrow{+4} 17, \dots$

The sequence does not have a common difference, so it is not arithmetic.

Problem 3

Think

How do you find the common ratio between two adjacent terms?
Divide each term by the previous term.

Identifying Geometric Sequences

Which of the following are geometric sequences?

A $20 \xrightarrow{\times 10} 200 \xrightarrow{\times 10} 2{,}000 \xrightarrow{\times 10} 20{,}000 \xrightarrow{\times 10} 200{,}000, \dots$

There is a common ratio, $r = 10$. So, the sequence is geometric.

B $2 \xrightarrow{\times 2} 4 \xrightarrow{\times 1.5} 6 \xrightarrow{\times 1.\overline{3}} 8 \xrightarrow{\times 1.25} 10, \dots$

There is no common ratio. So, the sequence is not geometric.

C $5 \xrightarrow{\times -1} -5 \xrightarrow{\times -1} 5 \xrightarrow{\times -1} -5 \xrightarrow{\times -1} 5, \dots$

There is a common ratio, $r = -1$. So, the sequence is geometric.

Problem 4

TEKS Process Standard (1)(A)

Writing an Explicit Formula for an Arithmetic Sequence

Online Auction **An online auction works as shown below. Write an explicit formula to represent the bids as an arithmetic sequence. What is the twelfth bid?**

Make a table of the bids. Identify the first term and common difference.

Term Number, n	1	2	3	4
Value of Term, $A(n)$	200	210	220	230

The first term $A(1)$ is 200.

+10 +10 +10

The common difference d is 10.

Plan

What information do you need to write a rule for an arithmetic sequence?
You need the first term of the sequence and the common difference.

Substitute $A(1) = 200$ and $d = 10$ into the formula $A(n) = A(1) + (n - 1)d$. The explicit formula $A(n) = 200 + (n - 1)10$ represents the arithmetic sequence of the auction bids. To find the twelfth bid, evaluate $A(n)$ for $n = 12$.

$$A(12) = 200 + (12 - 1)10 = 310$$

The twelfth bid is \$310.

Problem 5

TEKS Process Standard (1)(E)

Writing an Explicit Formula for a Geometric Sequence

Plan

What do you need to know in order to write an explicit formula for a geometric sequence?
You need the common ratio and the starting value.

A **Find the explicit formula for the sequence 7, 21, 63, 189, . . .**

The starting value $A(1)$ is 7. The common ratio r is $\frac{21}{7} = 3$.

$A(n) = A(1) \cdot r^{n-1}$ Use the formula.

$A(n) = 7 \cdot r^{n-1}$ Substitute the starting value for $A(1)$.

$A(n) = 7 \cdot 3^{n-1}$ Substitute the common ratio for r.

The explicit formula is $A(n) = 7 \cdot 3^{n-1}$.

B **Find the value of the 5th term of the sequence.**

$A(5) = 7 \cdot 3^{5-1}$ Use the explicit formula. Substitute 5 for n.

$= 7 \cdot 3^4$ Simplify.

$= 567$

The fifth term of the sequence is 567.

PRACTICE and APPLICATION EXERCISES

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Describe a pattern in each sequence. What are the next two terms of each sequence?

1. 3, 8, 13, 18, . . .

2. 4, -12, 36, -108, . . .

3. 8, 12, 18, 27, . . .

4. -8, -2, 4, 10, . . .

Tell whether the sequence is arithmetic. If it is, identify the common difference.

5. 6, 12, 18, 24, . . .

6. 1, 3, 6, 10, . . .

7. -5, -1, 5, 13, . . .

8. 2.4, 3.6, 4.8, 6.0, . . .

Tell whether the sequence is geometric. If it is, identify the common ratio.

9. 2, 4, 6, 8, . . .

10. 3, 6, 12, 24, . . .

11. 1, 4, 9, 16, . . .

12. 5, -10, 20, -40, . . .

Write an explicit formula for the *n*th term of each arithmetic sequence. Find the sixth and tenth terms of each sequence.

13. 7, 4.5, 2, -0.5, . . .

14. -1, 8, 17, 26, . . .

15. $A(1) = 12, d = \frac{1}{2}$

16. $A(1) = 4, d = -1.3$

Write an explicit formula for the *n*th term of each geometric sequence. Find the fifth and eighth terms of each sequence.

17. 9, -9, 9, -9, . . .

18. 3, 6, 12, 24, . . .

19. $A(1) = -16, r = \frac{1}{2}$

20. $A(1) = 2, r = 10$

Analyze Mathematical Relationships (1)(F) **Write the first six terms in each sequence. Explain what the fifth term means in the context of the situation.**

21. The population of a community is 35,000 in January and increases by 500 each month thereafter.

22. A department store is having a sale on discontinued items. One jacket starts at an initial price of \$48. Its price is reduced by 25% each week until it sells. Round each term to the nearest hundredth.

23. Best Value is having a sale on snack crackers. Buy the first box for \$3. Each additional box is \$1.75.

Create Representations to Communicate Mathematical Ideas(1)(E) **Write an explicit formula for each sequence described. Use the formula to find $A(7)$.**

24. There are two different geometric sequences with $A(1) = 5$ and $A(3) = 8$. Write an explicit formula for each sequence.

25. An arithmetic sequence contains the terms $A(2) = 4$ and $A(5) = 7$. Write an explicit formula for this sequence.

26. **Apply Mathematics (1)(A)** You open a bank account with \$200. Every year, you will earn 0.5% in interest on your account. You can use a geometric sequence to record the amount in the account each year.

a. At the end of each year, your account will contain the amount of money on deposit from the previous year plus the interest earned that year. What is the common ratio for the geometric sequence?

b. Write an explicit formula for the geometric sequence.

c. How much money will be in the account after 6 years?

Find the second, fourth, and eleventh terms of the sequence described by each explicit formula.

27. $A(n) = 5 + (n - 1)(-3)$

28. $A(n) = -3 + (n - 1)(5)$

29. $A(n) = 11 \cdot (2)^{n-1}$

30. $A(n) = 6 \cdot (3)^{n-1}$

TEXAS End-of-Course PRACTICE

31. Which of the following is an arithmetic sequence?

A. 2, 3, 5, 8, . . .

B. 2, 4, 6, 8, . . .

C. 2, 4, 8, 16, . . .

D. 2, −1, 5, −4, . . .

32. Which formula explicitly defines the sequence 4, 6, 9, $\frac{27}{2}$, . . .?

F. $A(n) = 4 \cdot \left(\frac{3}{2}\right)^{n-1}$

G. $A(n) = \frac{3}{2} \cdot (4)^{n-1}$

H. $A(n) = 4 + (n - 1)\frac{3}{2}$

J. $A(n) = 4 + (n - 1)2$

33. The nth term of an arithmetic sequence is defined by the formula $A(n) = -8 + (n - 1)3$. What is the eighth term in the sequence?

A. 11 **B.** 13 **C.** 15 **D.** 17

34. Write an explicit formula for the nth term of the geometric sequence with $A(1) = 5$ and $A(6) = 160$.

6-2 Arithmetic Sequences in Recursive Form

TEKS FOCUS

TEKS (12)(C) Identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(C), (1)(D), (12)(B), (12)(D)

VOCABULARY

- **Recursive formula** – A recursive formula defines the terms in a sequence by relating each term to the ones before it.
- **Recursive definition** – A recursive definition of a sequence has two parts, an initial condition and a recursive formula.

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

ESSENTIAL UNDERSTANDING

A sequence is a function whose domain is the natural numbers, and whose outputs are the terms of a sequence. You can write an arithmetic sequence using a recursive formula.

take note Key Concept Recursive Definition – Arithmetic Sequence

A **recursive formula** is a function rule that relates each term of a sequence after the first to the ones before it. Consider the sequence 7, 11, 15, 19, ... You can use the common difference of the terms of an arithmetic sequence to write a recursive formula for the sequence. For the sequence 7, 11, 15, 19, . . . , the common difference is 4.

Let n = the term number in the sequence.
Let $A(n)$ = the value of the nth term of the sequence.

value of term 1 $= A(1) = 7$

value of term 2 $= A(2) = A(1) + 4 = 11$

value of term 3 $= A(3) = A(2) + 4 = 15$

value of term 4 $= A(4) = A(3) + 4 = 19$

value of term $n = A(n) = A(n-1) + 4$

The common difference is 4.

The value of the previous term plus 4

The recursive formula for the arithmetic sequence above is $A(n) = A(n-1) + 4$. The recursive formula together with the starting value $A(1) = 7$ defines the sequence.

A general **recursive definition** for an arithmetic sequence with common difference d has two parts:

$A(1) =$ first term	Initial condition, or starting value
$A(n) = A(n-1) + d$, for $n \geq 2$	Recursive formula

Problem 1

TEKS Process Standard (1)(D)

Writing a Recursive Formula

Write a recursive formula for the arithmetic sequence below. What is the value of the 8th term?

$$70, \quad 77, \quad 84, \quad 91, \ldots$$

$$+7 \quad +7 \quad +7$$

Step 1 $A(1) = 70$ — First term of the sequence

$A(2) = A(1) + 7 = 70 + 7 = 77$ — $A(2)$ is found by adding 7 to $A(1)$.

$A(3) = A(2) + 7 = 77 + 7 = 84$ — $A(3)$ is found by adding 7 to $A(2)$.

$A(4) = A(3) + 7 = 84 + 7 = 91$ — $A(4)$ is found by adding 7 to $A(3)$.

$A(n) = A(n-1) + 7$ — $A(n)$ is found by adding 7 to $A(n-1)$.

The recursive formula for the arithmetic sequence is $A(n) = A(n-1) + 7$. The recursive formula together with the starting value $A(1) = 70$ defines the sequence.

Think

Does the recursive formula alone define the sequence?
No; you also need to know the starting value. Using this formula with a different starting value would produce a different sequence.

Step 2 To find the value of the 8th term, you need to extend the pattern.

$A(5) = A(4) + 7 = 91 + 7 = 98$

$A(6) = A(5) + 7 = 98 + 7 = 105$

$A(7) = A(6) + 7 = 105 + 7 = 112$

$A(8) = A(7) + 7 = 112 + 7 = 119$

The value of the 8th term is 119.

Problem 2

TEKS Process Standard (1)(C)

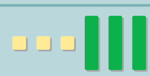
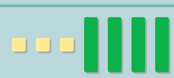
Identifying Terms of Recursive Arithmetic Sequences

Use the recursive definition $A(n) = A(n-1) + 10$ when $A(1) = 3$ to find the first five terms of the arithmetic sequence it defines.

Plan

What technique could you use to solve this problem?
You can solve this problem using mental math or manipulatives.

The sequence says that 10 is added to each previous term of the sequence and $A(1)$ is 3.

Term	Mental Math	Manipulatives	Solution
$A(1)$	$A(1) = 3$		3
$A(2)$	Think: $3 + 10$		13
$A(3)$	Think: $13 + 10$		23
$A(4)$	Think: $23 + 10$		33
$A(5)$	Think: $33 + 10$		43

Problem 3

Writing an Explicit Formula From a Recursive Formula

An arithmetic sequence is represented by the recursive formula $A(n) = A(n-1) + 12$. If the first term of the sequence is 19, write the explicit formula.

The first term is 19, so $A(1) = 19$.

$A(n) = A(n-1) + 12$ — Adding 12 to the previous term means that the common difference d is 12.

$A(n) = A(1) + (n-1)d$ — General form of an explicit formula

$A(n) = 19 + (n-1)12$ — Substitute 19 for $A(1)$ and 12 for d.

The explicit formula $A(n) = 19 + (n-1)12$ represents the arithmetic sequence.

Problem 4

TEKS Process Standard (1)(G)

Writing a Recursive Formula From an Explicit Formula

Think

What information does the explicit formula give you? You can use the explicit formula to identify the first term and the common difference.

An arithmetic sequence is represented by the explicit formula $A(n) = 32 + (n-1)(22)$. What is the recursive formula?

$A(n) = 32 + (n-1)(22)$ — 32 is the first term. 22 is the common difference.

A recursive formula relates the value of the term to the previous term using the common difference. Use $A(n)$ to represent the value of the term and $A(n-1)$ to represent the value of the previous term.

The arithmetic sequence is represented by the recursive formula $A(n) = A(n-1) + 22$. The recursive formula together with the starting value $A(1) = 32$ defines the sequence.

PRACTICE and APPLICATION EXERCISES

ONLINE HOMEWORK

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1. Select a tool or technique such as manipulatives or mental math to find $A(7)$ when $A(n) = A(n-1) + 5$ and $A(1) = -10$.
2. Select a tool or technique such as manipulatives or mental math to find $A(6)$ when $A(n) = A(n-1) + 2$ and $A(1) = 6$.
3. Use the recursive definition $A(n) = A(n-1) - 4$, when $A(1) = 10$ to find the first five terms of the arithmetic sequence it defines.
4. **Apply Mathematics (1)(A)** A runner is training for a marathon. The first week, she runs 2 miles during each training session. Each week, she increases her distance by 1.5 miles. Write a recursive definition for this sequence and use this definition to find her training distance in the tenth week.

Write a recursive definition for each sequence.

5. 1.1, 1.9, 2.7, 3.5, . . . **6.** 99, 88, 77, 66, . . . **7.** 23, 38, 53, 68, . . .

Write an explicit formula for each recursive formula.

8. $A(n) = A(n-1) + 12; A(1) = 12$ **9.** $A(n) = A(n-1) + 3.4; A(1) = 7.3$

10. $A(n) = A(n-1) + 3; A(1) = 6$ **11.** $A(n) = A(n-1) - 0.3; A(1) = 0.3$

Write a recursive definition for each explicit formula.

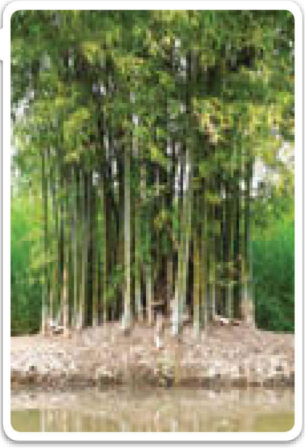
12. $A(n) = 5 + (n-1)(3)$ **13.** $A(n) = 3 + (n-1)(-5)$

14. $A(n) = -1 + (n-1)(-2)$ **15.** $A(n) = 4 + (n-1)(1)$

16. **Explain Mathematical Ideas (1)(G)** An arithmetic sequence can be represented by the explicit formula $A(n) = -10 + (n-1)(4)$. Describe the relationship between the first term and the second term and between the second term and the third term. Write a recursive definition to represent this sequence.

17. Write a recursive definition for a sequence that has 25 as the sixth term.

Write the first six terms in each sequence. Explain what the sixth term means in the context of the situation.



18. A cane of bamboo is 30 in. tall the first week and grows 6 in. per week thereafter.

19. You borrow \$350 from a friend the first week and pay the friend back \$25 each week thereafter.

20. **Explain Mathematical Ideas (1)(G)** Suppose the first Friday of a new year is the fourth day of that year. Will the year have 53 Fridays regardless of whether or not it is a leap year? Explain.

21. **Apply Mathematics (1)(A)** Buses run every 9 min starting at 6:00 A.M. You get to the bus stop at 7:16 A.M. How long will you wait for a bus?

22. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** Use the table below that shows an arithmetic sequence. Complete the table and then graph the ordered pairs (x, y) on a coordinate plane. What do you notice about the points on your graph?

x	1	2	3	4
y	5	8	■	■

Using the recursive definition for each arithmetic sequence, find the second, third, and fourth terms of the sequence. Then write the explicit formula that represents the sequence.

23. $A(n) = A(n-1) - 4; A(1) = 8$ **24.** $A(n) = A(n-1) + 1.2; A(1) = 8.8$

25. $A(n) = A(n-1) + 3; A(1) = 13$ **26.** $A(n) = A(n-1) - 2; A(1) = 0$

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive definition and an explicit formula to represent it.

27. 0.3, 0.9, 1.5, 2.1, . . .

28. $-3, -7, -11, -15, \ldots$

29. 1, 8, 27, 64, . . .

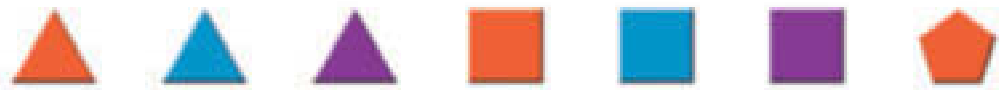
30. $-5, 5, -5, 5, \ldots$

Find the common difference of each arithmetic sequence. Then find the next term.

31. $4, x + 4, 2x + 4, 3x + 4, \ldots$

32. $a + b + c, 4a + 3b + c, 7a + 5b + c, \ldots$

33. a. Draw the next figure in the pattern.



b. What is the color of the twentieth figure? Explain.

c. How many sides does the twenty-third figure have? Explain.

TEXAS End-of-Course PRACTICE

34. What is the seventh term of the arithmetic sequence represented by the function $A(n) = -9 + (n - 1)(0.5)$?

A. -7

B. -6.5

C. -6

D. -5.5

35. What is the solution of $-24 + s > 38$?

F. $s < 14$

G. $s > 14$

H. $s < 62$

J. $s > 62$

36. Marta's starting annual salary is $26,500. At the beginning of each new year, she receives a $2880 raise. Write a recursive definition to find Marta's salary for the nth year. What will Marta's salary be for her sixth year?

6-3 Geometric Sequences in Recursive Form

TEKS FOCUS

TEKS (12)(C) Identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(A), (12)(B), (12)(D)

VOCABULARY

- **Number sense** – the understanding of what numbers mean and how they are related

ESSENTIAL UNDERSTANDING

In a geometric sequence, the ratio of any term to its preceding term is a constant value.

take note

Key Concept Recursive Definition – Geometric Sequence

A geometric sequence with a starting value a and a common ratio r is a sequence of the form

$a, ar, ar^2, ar^3, \ldots$

Consider the geometric sequence 4, 20, 100, 500, ... You can use the common ratio of the terms of a geometric sequence to write a recursive formula. For the sequence 4, 20, 100, 500, . . . , the common ratio is 5.

value of term 1 $= A(1) = 4$

The common ratio is 5.

value of term 2 $= A(2) = A(1) \cdot 5 = 20$

value of term 3 $= A(3) = A(2) \cdot 5 = 100$

The value of the previous term times 5

value of term 4 $= A(4) = A(3) \cdot 5 = 500$

value of term $n = A(n) = A(n-1) \cdot 5$

The recursive formula for the geometric sequence above is $A(n) = A(n-1) \cdot 5$.

The recursive formula together with the starting value $A(1) = 4$ defines the sequence.

A general recursive definition for a geometric sequence with common ratio r has two parts:

$A(1) = a$	Initial condition, or starting value
$A(n) = A(n-1) \cdot r$, for $n \geq 2$	Recursive formula

Problem 1

Plan

What do you need to know in order to write a recursive formula for a geometric sequence?
You need the common ratio and the starting value.

Writing a Recursive Formula for a Geometric Sequence

Find the recursive formula for the sequence 7, 21, 63, 189, ...

The starting value $A(1)$ is 7. The common ratio r is $\frac{21}{7} = 3$.

$A(1) = a;\ A(n) = A(n-1) \cdot r$	Use the formula.
$A(1) = 7;\ A(n) = A(n-1) \cdot r$	Substitute the starting value for $A(1)$.
$A(1) = 7;\ A(n) = A(n-1) \cdot 3$	Substitute the common ratio for r.

The recursive formula is $A(n) = A(n-1) \cdot 3$. The recursive formula together with the starting value $A(n) = 7$ defines the sequence.

Problem 2

TEKS Process Standard (1)(C)

Identifying Terms of Recursive Geometric Sequences

Consider the following geometric sequences:

Sequence A	**Sequence B**
$A(1) = 1$	$B(1) = 10$
$A(n) = \frac{1}{2} \cdot A(n-1)$	$B(n) = \frac{1}{10} \cdot B(n-1)$

Which is greater $A(100)$ or $B(100)$?

Think

Should you calculate $A(100)$ and $B(100)$ exactly?
No. It can be very labor intensive to use recursive definitions for such large inputs. There are more efficient ways to compare these terms.

Method 1 Use real objects to help represent the values of the terms in the sequences.

You can represent $A(1)$ with a piece of string. Since $A(1) = 1$ and $B(1) = 10$, use a string 10 times as long to represent $B(1)$. Because the common ratio of each sequence is less than one, you can represent the next term of each sequence by cutting off a section of string.

After the first cut to each string, the string representing $B(2)$ will be longer than the string representing $A(2)$. After the second cut, the string representing $A(3)$ will be longer than the string representing $B(3)$.

In every cut after the second cut, the string representing terms in Sequence B will be shorter than the strings representing terms in Sequence A. This is because you cut away a greater portion of the string for Sequence A than you do for Sequence B.

So $A(100)$ is greater than $B(100)$.

continued on next page ▶

Problem 2 *continued*

Method 2 Use number sense.

Consider the first three terms of each sequence.

Sequence B starts with a greater value.

$A(1) = 1$

$A(2) = \frac{1}{2} \cdot 1 = \frac{1}{2}$

$A(3) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$B(1) = 10$

$B(2) = \frac{1}{10} \cdot 10 = 1$

$B(3) = \frac{1}{10} \cdot 1 = \frac{1}{10}$

Since the common ratio for Sequence B, $\frac{1}{10}$, is smaller than the common ratio for Sequence A, $\frac{1}{2}$, Sequence B gets smaller faster than Sequence A.

Note that while Sequence B starts out greater than Sequence A, by the third term, Sequence B is less than Sequence A. And since Sequence B gets smaller faster than Sequence A, terms of Sequence B are less than the corresponding terms of Sequence A for every term after the third term. So $A(100)$ is greater than $B(100)$.

Problem 3

TEKS Process Standard (1)(A)

Using Sequences

A manager at a clothing store created sequences to show the original price and the marked-down prices of jeans and sweaters in two different ways. Write a recursive formula and an explicit formula for each sequence. What will the price of each item be after the 6th markdown for each sequence?

Think

Which term number represents the 6th markdown?
The 1st term represents the original price, the 2nd term represents the 1st markdown, the 3rd term represents the 2nd markdown, and so on. Therefore, the 7th term represents the 6th markdown.

First Sequence (Jeans)		Second Sequence (Sweaters)
\$60, \$51, \$43.35, \$36.85, . . .		**\$60, \$52, \$44, \$36, . . .**
common ratio = 0.85		**common difference = −8**
$A(1) = 60$		$A(1) = 60$
$A(n) = A(n-1) \cdot 0.85$	Recursive formula	$A(n) = A(n-1) - 8$
$A(n) = 60 \cdot 0.85^{n-1}$	Explicit formula	$A(n) = 60 + (n-1)(-8)$
$A(7) = 60 \cdot 0.85^{7-1}$	Substitute 7 for *n*.	$A(7) = 60 + (7-1)(-8)$
$A(7) \approx \$22.63$	Simplify.	$A(7) = \$12$

Continuing the first sequence, the price of the jeans is \$22.63 after the 6th markdown.

Continuing the second sequence, the price of the sweaters is \$12 after the 6th markdown.

PRACTICE and APPLICATION EXERCISES

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Write the recursive definition for each geometric sequence.

1. 4, 8, 16, 32, . . .
2. 1, 5, 25, 125, . . .
3. 100, 50, 25, 12.5, . . .
4. 2, −8, 32, −128, . . .
5. $-\frac{1}{36}, \frac{1}{12}, -\frac{1}{4}, \frac{3}{4}, \ldots$
6. $192, 128, 85\frac{1}{3}, 56\frac{8}{9}, \ldots$

Find the indicated term of the geometric sequence.

7. $A(n) = \frac{3}{4} \cdot A(n-1)$, where $A(1) = 256$; $A(4)$

8. $A(n) = -1.5 \cdot A(n-1)$, where $A(1) = 10$; $A(3)$

9. **Select Tools to Solve Problems (1)(C)** Sequence A is $A(n) = 2 \cdot A(n-1)$, where $A(1) = 5$. Sequence B is $B(n) = 10 \cdot B(n-1)$, where $B(1) = 1$. Select a tool such as real objects, manipulatives, or paper and pencil to help you determine which sequence has the greater hundredth term.

10. **Select Techniques to Solve Problems (1)(C)** Sequence A is $A(n) = \frac{1}{4} \cdot A(n-1)$, where $A(1) = 1$. Sequence B is $B(n) = \frac{1}{15} \cdot B(n-1)$, where $B(1) = 15$. Select a technique such as mental math, estimation, or number sense to help you determine which sequence has the greater hundredth term.

11. A researcher begins a study with 400 mg of a certain radioactive material. The material has a half-life of 1 day, meaning that after 24 hours one half of the atoms in the sample have disintegrated. The amount in the sample after n days is given by the recursive formula $A(n) = \frac{1}{2} \cdot A(n-1)$. How much of the material will be left in the sample after 7 days?

12. **Apply Mathematics (1)(A)** A store manager plans to offer discounts on some sweaters according to this sequence: \$48, \$36, \$27, \$20.25, ... Write the explicit formula and recursive definition for the sequence.

13. A geometric sequence has an initial value of 18 and a common ratio of $\frac{1}{2}$. Write a recursive definition to represent this sequence.

14. Write the recursive definition that represents the sequence shown in the table.

n	1	2	3	4
$A(n)$	8	16	32	64

15. Write a geometric sequence. Then write the explicit formula and recursive definition for your sequence.

16. **Apply Mathematics (1)(A)** A certain culture of yeast increases by 50% every three hours. A scientist places 9 grams of the yeast in a culture dish. Write the explicit formula and recursive definition for the geometric sequence formed by the growth of the yeast.

17. The differences between consecutive terms in a geometric sequence form a new geometric sequence. For instance, when you take the differences between the consecutive terms of the geometric sequence 5, 15, 45, 135, . . . , you get $15 - 5$, $45 - 15$, $135 - 45$, ... The new geometric sequence is 10, 30, 90, ... Compare the two sequences. How are they similar, and how do they differ?

Determine if each sequence is a geometric sequence. If it is, find the common ratio and write the explicit formula and recursive definition.

18. 5, 10, 20, 40, . . .

19. 20, 15, 10, 5, . . .

20. 3, −9, 27, −81, . . .

21. 98, 14, 2, $\frac{2}{7}$, . . .

22. −3, −1, 1, 3, . . .

23. 200, −100, 50, −25, . . .

Identify each sequence as *arithmetic, geometric,* or *neither*.

24. −4, 1, 6, 11, . . .

25. 1, 2, 3, 5, . . .

26. 2, 8, 32, 128, . . .

27. Explain Mathematical Ideas (1)(G) Suppose you are rehearsing for a concert. You plan to rehearse the piece you will perform four times the first day and then to double the number of times you rehearse the piece each day until the concert. What are two formulas you can write to describe the sequence of how many times you will rehearse the piece each day?

TEXAS End-of-Course PRACTICE

28. Which of the following is the explicit formula for the geometric sequence 15, 3, 0.6, 0.12, . . .?

A. $A(n) = 15 \cdot 0.2^{n-1}$

B. $A(1) = 0.12; A(n) = 5 \cdot A(n-1)$

C. $A(1) = 15; A(n) = 0.2 \cdot A(n-1)$

D. $A(n) = 0.2 \cdot 15^{n-1}$

29. Which of the following is the recursive definition for the geometric sequence 2, 12, 72, 432, . . .?

F. $A(n) = 2 \cdot 6^{n-1}$

G. $A(1) = 2; A(n) = 6 \cdot A(n-1)$

H. $A(1) = 6; A(n) = 2 \cdot A(n-1)$

J. $A(n) = 6 \cdot 2^{n-1}$

30. Which of the following is the explicit formula for the geometric sequence 12, 18, 27, 40.5, . . .?

A. $A(n) = 15 \cdot 1.2^{n-1}$

B. $A(1) = 12; A(n) = 0.5 \cdot A(n-1)$

C. $A(1) = 12; A(n) = 1.5 \cdot A(n-1)$

D. $A(n) = 12 \cdot 1.5^{n-1}$

31. Which of the following is both an arithmetic sequence and a geometric sequence?

F. 1, −1, 1, −1, . . .

G. 16, 24, 36, 54, . . .

H. 1, 4, 9, 16, . . .

J. 5, 5, 5, 5, . . .

32. Write the explicit formula and recursive definition for the geometric sequence 27, 36, 48, 64, ...

Topic 6 Review

TOPIC VOCABULARY

- arithmetic sequence, *p. 246*
- common difference, *p. 246*
- common ratio, *p. 246*
- explicit formula, *p. 246*
- geometric sequence, *p. 246*
- recursive definition, *p. 251*
- recursive formula, *p. 251*
- sequence, *p. 246*
- term of a sequence, *p. 246*

Check Your Understanding

Choose the vocabulary term that correctly completes the sentence.

1. A(n) __?__ is a number sequence that has a common ratio between terms.

2. Any number in a sequence is called a(n) __?__.

3. The fixed number in an arithmetic sequence is called the __?__.

6-1 Arithmetic and Geometric Sequences

Quick Review

A **sequence** is an ordered list of numbers, called terms, that often forms a pattern.

In an **arithmetic sequence**, the difference between terms is constant and is called the **common difference**.

In a **geometric sequence**, the ratio of any term to its preceding term is constant and is called the **common ratio**.

Example

Write an explicit formula for each sequence.

35, 31, 27, 23, . . .

- This is an arithmetic sequence. The pattern of this sequence is "add -4 to the previous term."
- The common difference is -4.
- The explicit formula for this sequence is $A(n) = 35 + (n - 1)(-4)$.

3, 30, 300, 3000, . . .

- This is a geometric sequence. The pattern of this sequence is "multiply the previous term by 10."
- The common ratio is 10.
- The explicit formula for this sequence is $A(n) = 3 \cdot (10)^{n-1}$.

Exercises

Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Identify the common difference of any arithmetic sequence and the common ratio of any geometric sequence.

4. 800, 400, 200, 100, . . .

5. 800, 700, 500, 200, . . .

6. 800, 725, 650, 575, . . .

Write an explicit formula for each sequence.

7. $-1, 3, -9, 27, \ldots$

8. $-17, -11, -5, 1, \ldots$

Write an explicit formula for each sequence. Use the formula to find the tenth term in the sequence.

9. arithmetic: $A(1) = 11, d = -2$

10. geometric: $A(1) = 11, r = -2$

6-2 Arithmetic Sequences in Recursive Form

Quick Review

An arithmetic sequence can be represented by a **recursive definition**.

Example

Write a recursive definition for the arithmetic sequence below.

4, 14, 24, 34, . . .

$A(1) = 4$

$A(2)$ is found by adding 10 to $A(1)$.

$A(3)$ is found by adding 10 to $A(2)$.

$A(4)$ is found by adding 10 to $A(3)$.

$A(n)$ is found by adding 10 to $A(n - 1)$.

The recursive definition for this sequence is $A(n) = A(n - 1) + 10$, where $A(1) = 4$.

Exercises

For each sequence, write a recursive definition.

11. 3, 8, 13, 18, . . .

12. $-2, -5, -8, -11, \dots$

13. 4, 6.5, 9, 11.5, . . .

14. $18, 11, 4, -3, \dots$

For each recursive definition, find an explicit formula that represents the same sequence.

15. $A(n) = A(n - 1) + 3$; $A(1) = 4$

16. $A(n) = A(n - 1) + 11$; $A(1) = 13$

17. $A(n) = A(n - 1) - 1$; $A(1) = 19$

6-3 Geometric Sequences in Recursive Form

Quick Review

A geometric sequence can be represented by a recursive definition.

Example

Find the common ratio of the geometric sequence.

$$2, \underset{\times 3}{\curvearrowright} 6, \underset{\times 3}{\curvearrowright} 18, \underset{\times 3}{\curvearrowright} 54, \dots$$

The common ratio of the geometric sequence is 3.

Write a recursive definition to represent the geometric sequence.

$$256, \underset{\times \frac{1}{4}}{\curvearrowright} 64, \underset{\times \frac{1}{4}}{\curvearrowright} 16, \underset{\times \frac{1}{4}}{\curvearrowright} 4, \dots$$

$A(1) = 256$; $A(n) = A(n - 1) \cdot \frac{1}{4}$

Exercises

Find the common ratio of each geometric sequence.

18. 10, 20, 40, 80, . . .

19. 1, 10, 100, 1000, . . .

20. 100, 20, 4, 0.8, . . .

21. 6561, 2187, 729, 243, . . .

Write a recursive definition to represent the geometric sequence.

22. 20, 60, 180, 540, . . .

23. 5, 2.5, 1.25, 0.625, . . .

24. 3, 12, 48, 192, . . .

25. 10, 1, 0.1, 0.01, . . .

Topic 6 TEKS Cumulative Practice

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. Tell whether the sequence is geometric. If it is, identify the common ratio.

25, 5, 1, 0.2, . . .

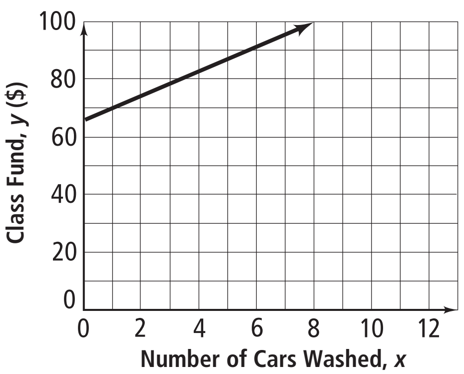
A. geometric sequence; $\frac{1}{5}$

B. geometric sequence; 5

C. geometric sequence; 20

D. not a geometric sequence

2. Hilo's class fund has \$65. The class is having a car wash to raise more money for a trip. The graph below models the amount of money the class will have if it charges \$4 for each car washed.



How would the graph change if the class charged \$5 per car washed?

F. The y-intercept would increase.

G. The slope would increase.

H. The y-intercept would decrease.

J. The slope would decrease.

3. A system has two linear equations in two variables. The graphs of the equations have the same slope but different y-intercepts. How many solutions does the system have?

A. 0 **C.** 2

B. 1 **D.** infinitely many

4. What is the range of the function $f(x) = 2x - 3$ for the domain $\{-4, -2, 0, 2, 4\}$?

F. $\{-11, -7, -3, 1, 5\}$

G. $\{1, 3, 5, 7, 11\}$

H. $\{-11, -7, -5, -3, -1\}$

J. $\{-11, -7, -3, 7, 11\}$

5. What is the domain of the function $f(x) = -4x$ for the range $\{-4, -2, 0, 2, 4\}$?

A. $\{-2, -1, 0, 1, 2\}$

B. $\{-4, -2, 0, 2, 4\}$

C. $\{-1, -0.75, 0, 0.75, 1\}$

D. $\{-1, -0.5, 0, 0.5, 1\}$

6. What is the fourth term of the sequence described by the explicit formula $A(n) = 2 + (n - 1)(-2.5)$?

F. $A(4) = -9.5$

G. $A(4) = -7.5$

H. $A(4) = -5.5$

J. $A(4) = 9.5$

7. What is the eleventh term of the sequence described by the explicit formula $A(n) = -9 + (n - 1)(3)$?

A. $A(11) = -39$

B. $A(11) = -21$

C. $A(11) = 21$

D. $A(11) = 39$

8. Solve the equation $3w + 2 = w - 4$.

F. $w = -3$

G. $w = -1$

H. $w = 1$

J. $w = 3$

9. Solve the equation $4(5n + 1) = 8(3n - 5)$.

A. $n = -11$

B. $n = 1.5$

C. $n = 9$

D. $n = 11$

10. A bottle holds 48 tsp of vanilla. The amount A of vanilla remaining in the bottle decreases by 2 tsp per batch b of cookies. What linear function rule best represents this situation?

F. $A = 48 - 2b$

G. $A = 48 + 2b$

H. $A = 2b - 48$

J. $A = 48 \div 2b$

11. You are buying party favors that cost $2.47 each. You can spend no more than $30 on the party favors. What linear inequality best represents this situation?

A. $2.47p < 30$

B. $2.47 \leq 30p$

C. $2.47p \leq 30$

D. $2.47p > 30$

12. Tell whether the sequence is arithmetic. If it is, identify the common difference.

4, 12, 20, 28, . . .

F. not an arithmetic sequence

G. arithmetic sequence; -8

H. arithmetic sequence; 8

J. arithmetic sequence; 3

13. Identify a recursive definition for the following geometric sequence.

10, 40, 160, 640, . . .

A. $A(1) = 10; A(n) = A(n - 1) \cdot \frac{1}{4}$

B. $A(1) = 10; A(n) = A(n - 1) \cdot 4$

C. $A(1) = 4; A(n) = A(n - 1) \cdot 10$

D. $A(1) = 10; A(n - 1) = A(n) \cdot 4$

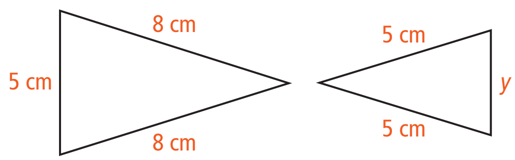
Gridded Response

14. The sum of two consecutive integers is -15. What is the product of the two integers?

15. Max writes a number pattern in which each number in the pattern is 1 less than twice the previous number. If the first number is 2, what is the fifth number?

16. In a bird sanctuary, 30% of the birds are hummingbirds. If there are about 350 birds in the sanctuary at any given time, how many are hummingbirds?

17. The figures below are similar. What is the missing length in centimeters?



18. In the late afternoon, a 3.5-ft child casts a 60-in. shadow. The child is next to a telephone pole that casts a 50-ft shadow, forming similar triangles. How tall is the telephone pole in feet?

19. The scale on a map is 1 in. : 25 mi. You measure 6.5 in. between two towns. What is the actual distance in miles?

Constructed Response

20. After one customer buys 4 new tires, a garage recycling bin has 20 tires in it. After another customer buys 4 new tires, the bin has 24 tires in it. Write an explicit formula to represent the number of tires in the bin as an arithmetic sequence. How many tires are in the bin after 9 customers buy all new tires?

21. You have a cafeteria card worth \$50. After you buy lunch on Monday, its value is \$46.75. After you buy lunch on Tuesday, its value is \$43.50. Write an explicit formula to represent the amount of money left on the card as an arithmetic sequence. What is the value of the card after you buy 12 lunches?

22. The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, ... After the first two numbers, each number is the sum of the two previous numbers. What is the next term of the sequence? The eleventh term of the sequence?

23. Tell whether the relationship should be represented by a *continuous* or *discrete* graph.

the price of turkey that sells for \$.89 per pound

24. Tell whether the relationship should be represented by a *continuous* or *discrete* graph.

the profit you make selling flowers at \$1.50 each when each flower costs you \$.80

25. Look at the sequence below.

$$-3.2, -2.4, -1.6, -0.8, \ldots$$

a. Tell whether the sequence is an arithmetic sequence.

b. List the next three terms in the sequence.

c. Write a recursive definition for the sequence.

d. Write an explicit formula for the sequence.

26. Look at the sequence below:

$$10, 50, 250, \ldots$$

a. Tell whether the sequence is a geometric sequence.

b. List the next three terms in the sequence.

c. Write a recursive definition for the sequence.

d. Write an explicit formula for the sequence.

27. Look at the sequence below described by the explicit formula.

$$A(n) = 2 + (n - 1)(-2.5)$$

a. Find the second term in the sequence.

b. Find the fourth term in the sequence.

c. Find the eleventh term in the sequence.

28. Look at the sequence below described by the explicit formula.

$$A(n) = -9 + (n - 1)(3)$$

a. Find the second term in the sequence.

b. Find the fourth term in the sequence.

c. Find the eleventh term in the sequence.

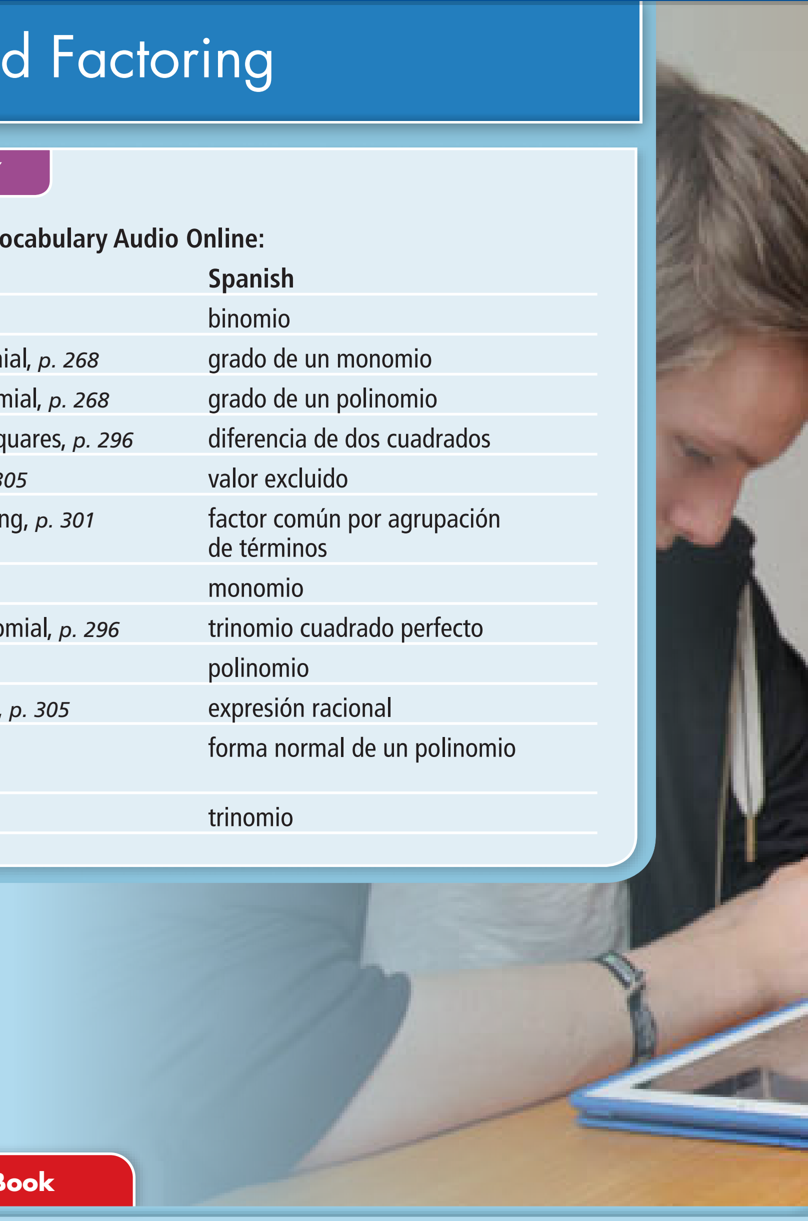
Topic 7 | Polynomials and Factoring

TOPIC OVERVIEW

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
binomial, *p. 268*	binomio
degree of a monomial, *p. 268*	grado de un monomio
degree of a polynomial, *p. 268*	grado de un polinomio
difference of two squares, *p. 296*	diferencia de dos cuadrados
excluded value, *p. 305*	valor excluido
factoring by grouping, *p. 301*	factor común por agrupación de términos
monomial, *p. 268*	monomio
perfect-square trinomial, *p. 296*	trinomio cuadrado perfecto
polynomial, *p. 268*	polinomio
rational expression, *p. 305*	expresión racional
standard form of a polynomial, *p. 268*	forma normal de un polinomio
trinomial, *p. 268*	trinomio



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3-Act Math

Who's Right?

People often approach a problem in different ways. Sometimes their solutions are the same, but other times different approaches lead to very different, but still valid solutions.

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7-1 Adding and Subtracting Polynomials

TEKS FOCUS

TEKS (10)(A) Add and subtract polynomials of degree one and degree two.

TEKS (1)(A) Apply mathematics to problems arising in everyday life, society, and the workplace.

Additional TEKS (1)(E)

VOCABULARY

- **Binomial** – a polynomial with two terms
- **Degree of a monomial** – the sum of the exponents of the variables in a nonzero monomial
- **Degree of a polynomial** – the highest degree of the monomials that make up a polynomial
- **Monomial** – a real number, a variable, or a product of a real number and one or more variables with whole-number exponents
- **Polynomial** – a monomial or a sum of monomials
- **Standard form of a polynomial** – a polynomial written so that the degrees of its monomial terms decrease from left to right
- **Trinomial** – a polynomial with three terms

- **Apply** – use knowledge or information for a specific purpose, such as solving a problem

ESSENTIAL UNDERSTANDING

You can use monomials to form larger expressions called *polynomials*. Polynomials can be added and subtracted.

take note

Key Concept Naming Polynomials

You can name a polynomial based on its degree or the number of monomials it contains.

Polynomial	Degree	Name Using Degree	Number of Terms	Name Using Number of Terms
6	0	Constant	1	Monomial
$5x + 9$	1	Linear	2	**Binomial**
$4x^2 + 7x + 3$	2	Quadratic	3	**Trinomial**
$2x^3$	3	Cubic	1	**Monomial**
$8x^4 - 2x^3 + 3x$	4	Fourth degree	3	Trinomial

Problem 1

Finding the Degree of a Monomial

Think

Why is the degree of a nonzero constant 0?
You can write a nonzero constant *c* as cx^0. The exponent is 0, so the degree is 0 also.

What is the degree of each monomial?

A $5x$ Degree: 1 $5x = 5x^1$. The exponent is 1.

B $6x^3y^2$ Degree: 5 The exponents are 3 and 2. Their sum is 5.

C 4 Degree: 0 $4 = 4x^0$. The degree of a nonzero constant is 0.

Problem 2

Adding and Subtracting Monomials

Think

Will the sum of two monomials always be a monomial?
No. The monomials must be like terms.

What is the sum or difference?

A $3x + 5x = (3 + 5)x$ Use the Distributive Property.
$= 8x$ Simplify.

B $4y - y = (4 - 1)y$ Use the Distributive Property.
$= 3y$ Simplify.

Problem 3

Classifying Polynomials

Think

Why do you need to combine like terms in part (B)?
To name a polynomial correctly based on its number of terms, you must first combine all like terms.

Write each polynomial in standard form. What is the name of the polynomial based on its degree and number of terms?

A $3x + 4x^2$
$4x^2 + 3x$ Place terms in order.
This is a quadratic binomial.

B $4x - 1 + 5x^3 + 7x$
$5x^3 + 4x + 7x - 1$ Place terms in order.
$5x^3 + 11x - 1$ Combine like terms.
This is a cubic trinomial.

Problem 4

TEKS Process Standard (1)(A)

Adding Polynomials

Travel **A researcher studied the number of overnight stays in U.S. National Park Service campgrounds and in the backcountry of the national park system over a 5-yr period. The researcher modeled the results, in thousands, with the following polynomials.**

Campgrounds: $-7.1x^2 - 180x + 5800$ **Backcountry:** $21x^2 - 140x + 1900$

In each polynomial, $x = 0$ corresponds to the first year in the 5-yr period. What polynomial models the total number of overnight stays in both types of sites?

continued on next page ▶

Problem 4 *continued*

Know
- Overnight stays in campgrounds: $-7.1x^2 - 180x + 5800$
- Overnight stays in backcountry: $21x^2 - 140x + 1900$

Need
A polynomial for the total number of overnight stays in campgrounds and backcountry

Plan
The word *both* implies addition, so add the two polynomials to represent the total.

Method 1 Add vertically. Line up like terms. Then add the coefficients.

$$\begin{array}{r} -7.1x^2 - 180x + 5800 \\ \underline{+\ 21x^2 - 140x + 1900} \\ 13.9x^2 - 320x + 7700 \end{array}$$

Method 2 Add horizontally. Group like terms. Then add the coefficients.

$$\begin{aligned} &(-7.1x^2 - 180x + 5800) + (21x^2 - 140x + 1900) \\ &= (-7.1x^2 + 21x^2) + (-180x - 140x) + (5800 + 1900) \\ &= 13.9x^2 - 320x + 7700 \end{aligned}$$

A polynomial that models the number of stays (in thousands) in campgrounds and backcountry over the 5-yr period is $13.9x^2 - 320x + 7700$.

Problem 5 TEKS Process Standard (1)(E)

Subtracting Polynomials

Simplify $(-3x^2 + 5x) - (5x^2 + 3x - 12)$.

Plan

How can Problem 4 help you solve this problem?
Change the subtraction to addition by adding the opposite. Then use the vertical or horizontal method from Problem 4 to add the polynomials.

Method 1 Subtract vertically.

$$\begin{array}{r} -3x^2 + 5x \\ \underline{-\ (5x^2 + 3x - 12)} \end{array}$$ Line up like terms.

$$\begin{array}{r} -3x^2 + 5x \\ \underline{-5x^2 - 3x + 12} \\ -8x^2 + 2x + 12 \end{array}$$ Then add the opposite of each term in the polynomial being subtracted.

Method 2 Subtract horizontally.

$(-3x^2 + 5x) - (5x^2 + 3x - 12)$

$= -3x^2 + 5x - 5x^2 - 3x + 12$ Write the opposite of each term in the polynomial being subtracted.

$= (-3x^2 - 5x^2) + (5x - 3x) + 12$ Group like terms.

$= -8x^2 + 2x + 12$ Simplify.

PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to **PearsonTEXAS.com**.

Find the degree of each monomial.

1. $3x$ **2.** $8a^3$ **3.** 20 **4.** $2b^8c^2$

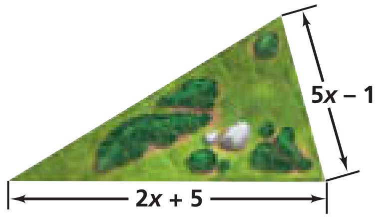
5. $-7y^3z$ **6.** -3 **7.** $12w^4$ **8.** 0

Simplify.

9. $\begin{array}{r} 5n^2 - 2 \\ \underline{-(3n^2 + 8)} \end{array}$ **10.** $\begin{array}{r} 6x^3 + 17 \\ \underline{-(4x^3 + 9)} \end{array}$ **11.** $\begin{array}{r} 2c^2 + 7c - 1 \\ \underline{-(c^2 - 10c + 4)} \end{array}$

12. $(14h^2 + 3h) - (9h^2 + 2h)$ **13.** $(-6w^4 + w^2) - (-2w^3 + 4w^2 - w)$

14. Apply Mathematics (1)(A) The perimeter of a triangular park is $16x + 3$. What is the missing length?



15. The perimeter of a trapezoid is $39a - 7$. Three sides have the following lengths: $9a$, $5a + 1$, and $17a - 6$. What is the length of the fourth side?

16. Explain Mathematical Ideas (1)(G) Describe and correct the error in finding the difference of the polynomials.

$$\begin{aligned}(4x^2 - x + 3) - (3x^2 - 5x - 6) &= 4x^2 - x + 3 - 3x^2 - 5x - 6 \\ &= 4x^2 - 3x^2 - x - 5x + 3 - 6 \\ &= x^2 - 6x - 3\end{aligned}$$

Simplify.

17. $12p + 8p$ **18.** $2m^3n^3 + 9m^3n^3$ **19.** $8w + w$

20. $3t^4 + 11t^4$ **21.** $x - 9x$ **22.** $30v^4w^3 - 12v^4w^3$

Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms.

23. $5y - 2y^2$ **24.** $-2q + 7$ **25.** $x^2 + 4 - 3x$

26. $6x^2 - 13x^2 - 4x + 4$ **27.** $c + 8c^3 - 3c^7$ **28.** $3z^4 - 5z - 2z^2$

Simplify.

29. $\begin{array}{r} 4w - 5 \\ \underline{+\ 9w + 2} \end{array}$ **30.** $\begin{array}{r} 6x^2 + 7 \\ \underline{+\ 3x^2 + 1} \end{array}$ **31.** $\begin{array}{r} 2k^2 - k + 3 \\ \underline{+\ 5k^2 + 3k - 7} \end{array}$

32. $(5x^2 + 3) + (15x^2 + 2)$ **33.** $(2g^4 - 3g + 9) + (-g^3 + 12g)$

34. a. Is the sum of two polynomials always a polynomial? Explain.

b. Is the difference of two polynomials always a polynomial? Explain.

35. **Apply Mathematics (1)(A)** The number of students at East High School and the number of students at Central High School over a 10-year period can be modeled by the following polynomials.

East High School: $-11x^2 + 133x + 1200$

Central High School: $-7x^2 + 95x + 1100$

In each polynomial, $x = 0$ corresponds to the first year in the 10-year period. What polynomial models the total number of students at both high schools?

Create Representations to Communicate Mathematical Ideas (1)(E) Simplify. Write each answer in standard form.

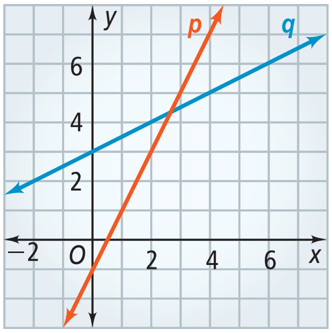
36. $(5x^2 - 3x + 7x) + (9x^2 + 2x^2 + 7x)$
37. $(y^3 - 4y^2 - 2) - (6y^3 + 4 - 6y^2)$
38. $(-9r^3 + 2r - 1) - (-5r^2 + r + 8)$
39. $(3z^3 - 4z + 7z^2) + (8z^2 - 6z - 5)$

40. **a.** Write the equations for each line. Use slope-intercept form.

b. Use your equations from part (a) to write a function for the difference between the y-coordinates $D(x)$ between points on lines p and q with the same x-value.

c. For what value of x does $D(x)$ equal zero?

d. **Analyze Mathematical Relationships (1)(F)** How does the x-value in part (c) relate to the graph?



Simplify each expression.

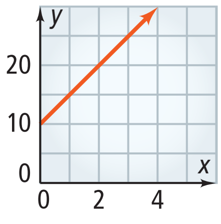
41. $(ab^2 + ba^3) + (4a^3b - ab^2 - 5ab)$
42. $(9pq^6 - 11p^4q) - (-5pq^6 + p^4q^4)$

TEXAS End-of-Course PRACTICE

43. What is a simpler form of $(3x^2 + 6x - 1) + (4x^2 + 5x + 9)$?

A. $-x^2 + x - 10$ **B.** $x^2 - x + 10$ **C.** $7x^2 + 11x + 8$ **D.** $7x^2 + 11x + 10$

44. The price of a gift basket of food can be modeled by a linear equation. You can use the graph at the right to find the price of the basket y, based on pounds of food x. What is the equation of the line?



F. $y = 5x + 10$ **H.** $y = x + 10$

G. $y = 10x + 5$ **J.** $y = 10x + 10$

45. Simplify $(8x^3 - 5x + 1) - (x^2 + 4)$. Show your work.

7-2 Multiplying and Factoring

TEKS FOCUS

TEKS (10)(D) Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(E), (10)(B)

VOCABULARY

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

ESSENTIAL UNDERSTANDING

You can use the Distributive Property to multiply a monomial by a polynomial.

take note

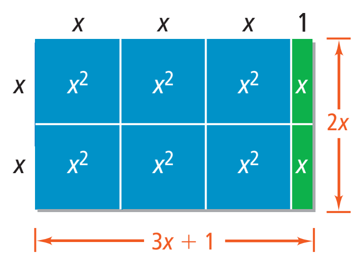
Key Concept Multiplying a Monomial by a Polynomial

You can multiply a monomial by a polynomial using the Distributive Property.

For example, consider the product $2x(3x + 1)$.

$$2x(3x + 1) = 2x(3x) + 2x(1)$$
$$= 6x^2 + 2x$$

You can show why the multiplication makes sense using the area model below.



Problem 1

Plan

What should you keep in mind when multiplying?
Remember to distribute $-x$ to *all* of the terms. Also remember to add the exponents instead of multiplying them.

Multiplying Polynomials of Degree One

Multiple Choice **What is a simpler form of $-x(9x + 7)$?**

(A) $-9x - 7$ (B) $9x^2 + 7x$ (C) $-9x^2 + 7$ (D) $-9x^2 - 7x$

$-x(9x + 7) = -x(9x) - x(7)$ Use the Distributive Property.

$= -9x^{1+1} - 7x$ Multiply coefficients and add exponents.

$= -9x^2 - 7x$ Simplify.

The correct answer is D.

Problem 2

TEKS Process Standard (1)(E)

Think

Why use the factors 5 and x to form the GCF, but not 3?
Both 5 and x are factors of *every* term of the polynomial, but 3 is only a factor of the last term.

Finding the Greatest Common Factor

What is the GCF of the terms of $5x^3 + 25x^2 + 45x$?

List the prime factors of each term. Identify the factors common to all terms.

$5x^3 = 5 \cdot x \cdot x \cdot x$

$25x^2 = 5 \cdot 5 \cdot x \cdot x$

$45x = 3 \cdot 3 \cdot 5 \cdot x$

Remember to list only the prime factors of the variables.

The GCF is $5 \cdot x$, or $5x$.

Problem 3

TEKS Process Standard (1)(G)

Factoring Out a Monomial

What is the factored form of $4x^2 - 24x$?

Think	Write
To factor the polynomial, first factor each term.	$4x^2 = 2 \cdot 2 \cdot x \cdot x$ $24x = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x$
Find the GCF of the two terms.	The GCF is $2 \cdot 2 \cdot x$, or $4x$.
Use the Distributive Property to factor out the GCF from each term. Then factor it out of the polynomial.	$4x^2 - 24x = 4x(x) + 4x(-6)$ $= 4x(x - 6)$ The factored form of the polynomial is $4x(x - 6)$.

Problem 4

Factoring a Polynomial Model

Plan

How can you find the shaded region's area? The shaded region is the entire square except for the circular portion. So, subtract the area of the circle from the area of the square.

Helipads **A helicopter landing pad, or helipad, is sometimes marked with a circle inside a square so that it is visible from the air. What is the area of the shaded region of the helipad at the right? Write your answer in factored form.**

Step 1 Find the area of the shaded region.

$A_1 = s^2$	Area of a square
$= (2x)^2$	Substitute $2x$ for s.
$= 4x^2$	Simplify.
$A_2 = \pi r^2$	Area of a circle
$= \pi x^2$	Substitute x for r.

The area of the shaded region is $A_1 - A_2$, or $4x^2 - \pi x^2$.

Step 2 Factor the expression.

First find the GCF.

$4x^2 = 2 \cdot 2 \cdot x \cdot x$

$\pi x^2 = \pi \cdot x \cdot x$

The GCF is $x \cdot x$, or x^2.

Step 3 Use the Distributive Property to factor out the GCF.

$$4x^2 - \pi x^2 = x^2(4) + x^2(-\pi)$$
$$= x^2(4 - \pi)$$

The factored form of the area of the shaded region is $x^2(4 - \pi)$.

PRACTICE and APPLICATION EXERCISES

ONLINE HOMEWORK

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Create Representations to Communicate Mathematical Ideas (1)(E) **Factor each polynomial.**

1. $t^2 + 8t$

2. $14n^2 - 35n + 28$

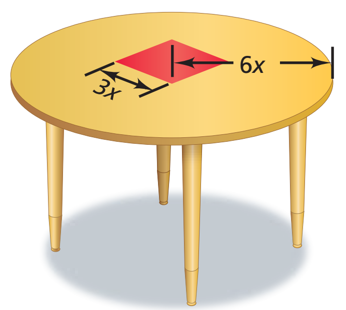
3. $g^4 + 24g^3 + 12g^2 + 4g$

4. $17xy^4 + 51x^2y^3$

5. $9m^4n^5 - 27m^2n^3$

6. $31a^6b^3 + 63a^5$

7. **Apply Mathematics (1)(A)** A circular table is painted yellow with a red square in the middle. The radius of the tabletop is $6x$. The side length of the red square is $3x$. What is the area of the yellow part of the tabletop? Write your answer in factored form.



8. **Apply Mathematics (1)(A)** A circular mirror is surrounded by a square metal frame. The radius of the mirror is $5x$. The side length of the metal frame is $15x$. What is the area of the metal frame? Write your answer in factored form.

Simplify. Write in standard form.

9. $-2x(5x^2 - 4x + 13)$ **10.** $-5y^2(-3y^3 + 8y)$ **11.** $10a(-6a^2 + 2a - 7)$

12. $p(p + 2) - 3p(p - 5)$ **13.** $t^2(t + 1) - t(2t^2 - 1)$ **14.** $3c(4c^2 - 5) - c(9c)$

15. **Apply Mathematics (1)(A)** A rectangular wooden frame has side lengths $5x$ and $7x + 1$. The rectangular opening for a picture has side lengths $3x$ and $5x$. What is the area of the wooden part of the frame? Write your answer in factored form.

16. **Explain Mathematical Ideas (1)(G)** Describe and correct the error made in multiplying.

$$\cancel{-3x(2x - 5) = -3x(2x) - 3x(5)}$$
$$\cancel{= -6x^2 - 15x}$$

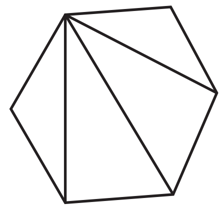
17. **a.** Factor $n^2 + n$.

b. **Justify Mathematical Arguments (1)(G)** Suppose n is an integer. Is $n^2 + n$ *always, sometimes,* or *never* an even integer? Justify your answer.

18. **a.** A polygon has n sides. How many diagonals will it have from one vertex?

b. The number of diagonals from all the vertices is $\frac{n}{2}(n - 3)$. Write this polynomial in standard form.

c. A polygon has 8 sides. How many diagonals does it have?



Simplify each product.

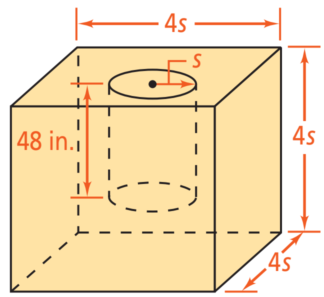
19. $7x(x + 4)$ **20.** $(b + 11)2b$ **21.** $3m(10 - m)$

Find the GCF of the terms of each polynomial.

22. $12x + 20$ **23.** $8w^2 - 18w$ **24.** $14z^4 - 42z^3 + 21z^2$

25. **Justify Mathematical Arguments (1)(G)** The GCF of two numbers p and q is 7. What is the GCF of p^2 and q^2? Justify your answer.

STEM **26.** **Apply Mathematics (1)(A)** The diagram shows a cube of metal with a cylinder cut out of it. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height.



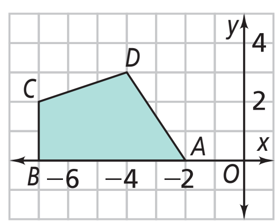
a. Write a formula in terms of s for the volume V of the metal left after the cylinder has been removed.

b. Factor your formula from part (a).

c. Find V in cubic inches for $s = 15$ in. Use $\pi = 3.14$.

TEXAS End-of-Course PRACTICE

27. What is the slope of the line that passes through $\overline{CD}$ in the graph?



28. Simplify the product $4x(5x^2 + 3x + 7)$. What is the coefficient of the x^2-term?

29. What is the solution of the equation $7x - 11 = 3$?

30. Simplify the product $8x^3(2x^2)$. What is the exponent?

7-3 Multiplying Binomials

TEKS FOCUS

TEKS (10)(B) Multiply polynomials of degree one and degree two.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(E), (10)(D)

VOCABULARY

- **Number sense** – the understanding of what numbers mean and how they are related

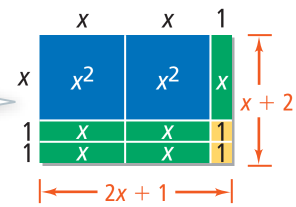
ESSENTIAL UNDERSTANDING

There are several ways to find the product of two binomials, including models, algebra, and tables.

take note Key Concept Using an Area Model

One way to find the product of two binomials is to use an area model, as shown at the right.

This model shows that $(2x + 1)(x + 2)$ can be written in standard form as $2x^2 + 5x + 2$.



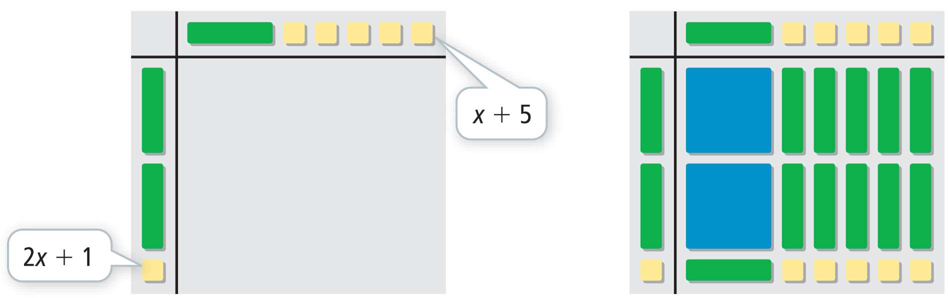
Problem 1

TEKS Process Standard (1)(C)

Multiplying Polynomials

Find the product $(2x + 1)(x + 5)$.

You can use algebra tiles to model the multiplication of two binomials.



$2x^2 + 10x + x + 5$ Identify the terms modeled by the tiles.

$2x^2 + 11x + 5$ Add coefficients of like terms.

Plan

How do you know which tiles to use?

Place a blue x^2 tile in the product when the factors are two green x tiles.

Place a green x tile in the product when the factors are a green x tile and a yellow unit tile.

Place a yellow unit tile in the product when the factors are two yellow unit tiles.

Problem 2

Using the Distributive Property

Plan

How can you use the Distributive Property with two binomials? Consider the second binomial as a single variable, and distribute it to each term of the first binomial.

What is the simplest form of $(2x + 4)(3x - 7)$?

$(2x + 4)(3x - 7) = 2x(3x - 7) + 4(3x - 7)$	Distribute the second factor, $3x - 7$.
$= 6x^2 - 14x + 4(3x - 7)$	Distribute $2x$.
$= 6x^2 - 14x + 12x - 28$	Distribute 4.
$= 6x^2 - 2x - 28$	Combine like terms.

Problem 3

TEKS Process Standard (1)(E)

Using a Table

What is the simplest form of $(x - 3)(4x - 5)$?

Know
Binomial factors

Need
Product of binomials written in standard form

Plan
Use a table.

When you use the Distributive Property to multiply binomials, notice that you multiply each term of the first binomial by each term of the second binomial. A table of products can help you organize your work.

Think

Is this the only table you can make? No. You can write the terms of $x - 3$ in a row and the terms of $4x - 5$ in a column.

	$4x$	-5
x	$4x^2$	$-5x$
-3	$-12x$	15

When labeling the rows and columns, think of $x - 3$ as $x + (-3)$. Think of $4x - 5$ as $4x + (-5)$.

The product is $4x^2 - 5x - 12x + 15$, or $4x^2 - 17x + 15$.

Problem 4

Using FOIL

Plan

How can a diagram help you multiply two binomials? Draw arrows from each term of the first binomial to each term of the second binomial. This will help you organize the products of the terms.

What is a simpler form of $(5x - 3)(2x + 1)$?

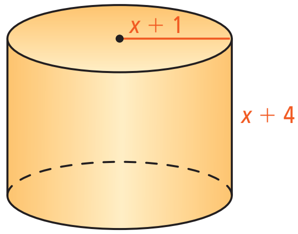
$$\begin{array}{ccccccccc} & & \text{First} & & \text{Outer} & & \text{Inner} & & \text{Last} \\ (5x - 3)(2x + 1) & = & (5x)(2x) & + & (5x)(1) & + & (-3)(2x) & + & (-3)(1) \\ & = & 10x^2 & + & 5x & - & 6x & - & 3 \\ & = & 10x^2 & - & x & - & 3 & & \end{array}$$

The product is $10x^2 - x - 3$.

Problem 5

Applying Multiplication of Binomials

Multiple Choice **A cylinder has the dimensions shown in the diagram. Which polynomial in standard form best describes the total surface area of the cylinder?**



(A) $2\pi x^2 + 4\pi x + 2\pi$

(B) $2\pi x^2 + 10\pi x + 8\pi$

(C) $4\pi x^2 + 14\pi x + 10\pi$

(D) $2\pi x^2 + 2\pi x + 10\pi$

The total surface area (S.A.) of a cylinder is given by the formula S.A. $= 2\pi r^2 + 2\pi rh$, where r is the radius of the cylinder and h is the height.

Think

How can you simplify $(x + 1)^2$?
Write the expression as $(x + 1)(x + 1)$ and multiply the binomials. You do not "distribute" the exponent to each term: $(x + 1)^2 \neq x^2 + 1^2$.

S.A. $= 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$= 2\pi(x + 1)^2 + 2\pi(x + 1)(x + 4)$	Substitute $x + 1$ for r and $x + 4$ for h.
$= 2\pi(x + 1)(x + 1) + 2\pi(x + 1)(x + 4)$	Write $(x + 1)^2$ as $(x + 1)(x + 1)$.
$= 2\pi\left(x^2 + x + x + 1\right) + 2\pi\left(x^2 + 4x + x + 4\right)$	Multiply binomials.
$= 2\pi\left(x^2 + 2x + 1\right) + 2\pi\left(x^2 + 5x + 4\right)$	Combine like terms.
$= 2\pi\left(x^2 + 2x + 1 + x^2 + 5x + 4\right)$	Factor out 2π.
$= 2\pi\left(2x^2 + 7x + 5\right)$	Combine like terms.
$= 4\pi x^2 + 14\pi x + 10\pi$	Write in standard form.

The correct answer is C.

Problem 6

Multiplying a Trinomial and a Binomial

What is the simplest form of $(3x^2 + x - 5)(2x - 7)$?

Plan

How should you align the polynomials?
Write the polynomials so that like terms are vertically aligned.

Multiply by arranging the polynomials vertically as shown.

$$\begin{array}{rrrr} & 3x^2 & +\,x & -\,5 \\ & & 2x & -\,7 \\ \hline & -21x^2 & -\,7x & +\,35 \\ 6x^3 & +\,2x^2 & -\,10x & \\ \hline 6x^3 & -\,19x^2 & -\,17x & +\,35 \end{array}$$

Multiply by -7.
Multiply by $2x$.
Add like terms.

The product is $6x^3 - 19x^2 - 17x + 35$.

PRACTICE and APPLICATION EXERCISES

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Select Tools to Solve Problems (1)(C) **Select and use a tool, such as algebra tiles or paper and pencil, to find each product.**

1. $(x - 3)(x + 2)$ **2.** $(4x + 1)(x + 4)$ **3.** $(2x + 5)(2x - 5)$

Simplify each product using the Distributive Property. Use a table to organize your work.

4. $(x + 5)(x - 4)$ **5.** $(2h - 7)(h + 9)$ **6.** $(3p + 4)(2p + 5)$

Simplify each product using the Distributive Property.

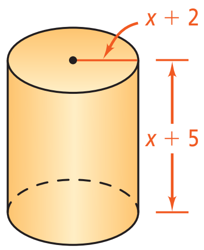
7. $(x + 7)(x + 4)$ **8.** $(2r - 3)(r + 1)$ **9.** $(2x + 7)(3x - 4)$

Simplify each product using the FOIL method.

10. $(a + 8)(a - 2)$ **11.** $(x + 4)(4x - 5)$ **12.** $(k - 6)(k + 8)$

13. $(3h + 2)(6h - 5)$ **14.** $(4w + 13)(w + 2)$ **15.** $(8c - 1)(6c - 7)$

16. Use Representations to Communicate Mathematical Ideas (1)(E) What is the total surface area of the cylinder? Write your answer as a polynomial in standard form.



17. Apply Mathematics (1)(A) The radius of a cylindrical gift box is $(2x + 3)$ in. The height of the gift box is twice the radius. What is the surface area of the cylinder? Write your answer as a polynomial in standard form.

Simplify each product.

18. $(x + 5)(x^2 - 3x + 1)$ **19.** $(k^2 - 4k + 3)(k - 2)$

20. $(2a^2 + 4a + 5)(5a - 4)$ **21.** $(2g + 7)(3g^2 - 5g + 2)$

22. Apply Mathematics (1)(A) A school's rectangular athletic fields currently have a length of 125 yd and a width of 75 yd. The school plans to expand both the length and the width of the fields by x yards. What polynomial in standard form represents the area of the expanded athletic field?

23. Apply Mathematics (1)(A) You are planning a rectangular dining pavilion. Its length is three times its width x. You want a stone walkway that is 3 ft wide around the pavilion. You have enough stones to cover 396 ft^2 and want to use them all in the walkway. What should the dimensions of the pavilion be?

Simplify each product. Write in standard form.

24. $(x^2 + 1)(x - 3)$

25. $(-n^2 - 1)(n + 3)$

26. $(b^2 - 1)(b^2 + 3)$

27. $(2m^2 + 1)(m + 5)$

28. $(c^2 - 4)(2c + 3)$

29. $(4z^2 + 1)(z + 3z^2)$

30. The dimensions of a rectangular prism are n, $n + 7$, and $n + 8$. Use the formula $V = \ell wh$ to write a polynomial in standard form for the volume of the prism.

31. Explain Mathematical Ideas (1)(G) Describe and correct the error made in finding the product.

~~$(x - 2)(3x + 4) = x(3x) + x(4) - 2(4)$~~
~~$= 3x^2 + 4x - 8$~~

32. Justify Mathematical Arguments (1)(G) Is the product of two polynomials always a polynomial? Explain.

33. Explain Mathematical Ideas (1)(G) Simplify each pair of products. What are the similarities between your two answers in each pair of products?

a. $(x + 1)(x + 1)$
$11 \cdot 11$

b. $(x + 1)(x + 2)$
$11 \cdot 12$

c. $(x + 1)(x + 3)$
$11 \cdot 13$

For Exercises 34–36, each expression represents the side length of a cube. Write a polynomial in standard form for the surface area of each cube.

34. $x + 2$

35. $3a + 1$

36. $2c^2 + 3$

37. Apply Mathematics (1)(A) Suppose you deposit \$1500 for college in a savings account that has an annual interest rate r (expressed as a decimal). At the end of 3 years, the value of your account will be $1500(1 + r)^3$ dollars.

a. Rewrite the expression $1500(1 + r)^3$ by finding the product $1500(1 + r)(1 + r)(1 + r)$. Write your answer in standard form.

b. How much money is in the account after 3 yr if the interest rate is 3% per year?

TEXAS End-of-Course PRACTICE

38. Which expression is equivalent to $(x + 4)(x - 9)$?

A. $x^2 + 5x - 36$ **B.** $x^2 - 5x - 36$ **C.** $x^2 - 13x - 36$ **D.** $x^2 - 13x - 5$

39. A trapezoid is determined by the following system of inequalities.

$y \geq 3$ $\quad y \leq 9$ $\quad x \leq 8$ $\quad y \leq 2x + 3$

a. Graph the trapezoid in the coordinate plane.

b. The formula for the area A of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 are the bases of the trapezoid and h is its height. What is the area of the trapezoid you graphed in part (a)? Show your work.

7-4 Multiplying Special Cases

TEKS FOCUS

TEKS (10)(B) Multiply polynomials of degree one and degree two.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(A), (1)(D)

VOCABULARY

- **Number sense** – the understanding of what numbers mean and how they are related

ESSENTIAL UNDERSTANDING

There are special rules you can use to simplify the square of a binomial or the product of a sum and difference.

take note

Key Concept The Square of a Binomial

Words The square of a binomial is the square of the first term plus twice the product of the two terms plus the square of the last term.

Algebra

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Examples

$(x + 4)^2 = x^2 + 8x + 16$

$(x - 3)^2 = x^2 - 6x + 9$

Key Concept The Product of a Sum and Difference

Words The product of the sum and difference of the same two terms is the difference of their squares.

Algebra

$(a + b)(a - b) = a^2 - b^2$

Examples

$(x + 2)(x - 2) = x^2 - 2^2 = x^2 - 4$

Problem 1

TEKS Process Standard (1)(D)

Squaring a Binomial

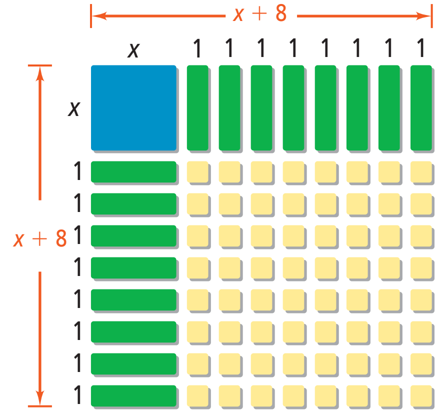
What is the simplest form of each product?

A $(x + 8)^2$

$(x + 8)^2 = x^2 + 2x(8) + 8^2$ Square the binomial.

$= x^2 + 16x + 64$ Simplify.

The area model at the right confirms the product is $x^2 + 16x + 64$.

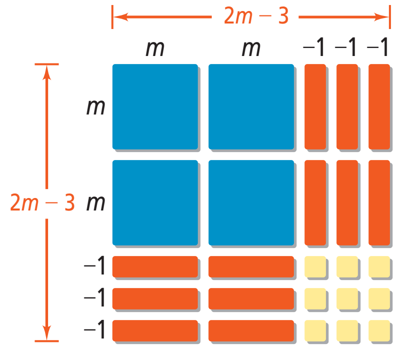


B $(2m - 3)^2$

$(2m - 3)^2 = (2m)^2 - 2(2m)(3) + 3^2$ Square the binomial.

$= 4m^2 - 12m + 9$ Simplify.

The area model at the right confirms the product is $4m^2 - 12m + 9$.



Problem 2

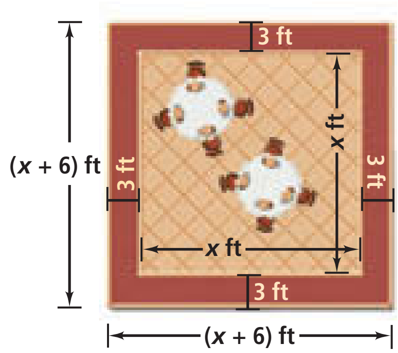
TEKS Process Standard (1)(A)

Applying Squares of Binomials

Plan

How do you find the area of the walkway? The area of the walkway is the difference of the total area and the area of the patio.

Exterior Design **A square outdoor patio is surrounded by a brick walkway, as shown. What is the area of the walkway?**



Step 1 Find the total area of the patio and walkway.

$(x + 6)^2 = x^2 + 2(x)(6) + 6^2$ Square the binomial.

$= x^2 + 12x + 36$ Simplify.

Step 2 Find the area of the patio.

The area of the patio is $x \cdot x$, or x^2.

Step 3 Find the area of the walkway.

Area of walkway = Total area − Area of patio

$= (x^2 + 12x + 36) - x^2$ Substitute.

$= (x^2 - x^2) + 12x + 36$ Group like terms.

$= 12x + 36$ Simplify.

The area of the walkway is $(12x + 36)$ ft^2.

Problem 3

Using Mental Math

What is 39^2? Use mental math.

Think

What number close to 39 can you square mentally?
The nearest multiple of 10 to 39 is 40, which is a number you should be able to square mentally.

$39^2 = (40 - 1)^2$	Write 39^2 as the square of a binomial.
$= 40^2 - 2(40)(1) + 1^2$	Square the binomial.
$= 1600 - 80 + 1$	Simplify.
$= 1521$	Simplify.

Problem 4

Finding the Product of a Sum and Difference

What is the simplest form of $(x^2 + 8)(x^2 - 8)$?

Plan

How do you choose which rule to use?
The first factor in the product is the sum of x^2 and 8. The second factor is the difference of x^2 and 8. So, use the rule for the product of a sum and difference.

Think	Write
Write the original product.	$(x^2 + 8)(x^2 - 8)$
Identify which terms correspond to *a* and *b* in the rule for the product of a sum and difference.	$a = x^2$; $b = 8$ $(a + b)(a - b) = a^2 - b^2$
Substitute for *a* and *b* in the rule.	$(x^2 + 8)(x^2 - 8) = (x^2)^2 - (8)^2$
Simplify.	$= x^4 - 64$

Problem 5

TEKS Process Standard (1)(C)

Finding a Product Using a Sum and a Difference

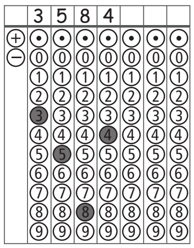
What is 64 • 56?

Think

How can you write 64 • 56 as the product of a sum and difference?
Find the number halfway between the factors. 60 is 4 units from each factor. Write the factors in terms of 60 and 4.

You can use mental math to find the product. Write the product as the product of a sum and a difference that you can rewrite as the difference of squares.

$64 \cdot 56 = (60 + 4)(60 - 4)$	Write as a product of a sum and a difference.
$= 60^2 - 4^2$	Use $(a + b)(a - b) = a^2 - b^2$.
$= 3600 - 16$	Simplify powers.
$= 3584$	Simplify.



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Use Multiple Representations to Communicate Mathematical Ideas (1)(D)
Simplify each expression.

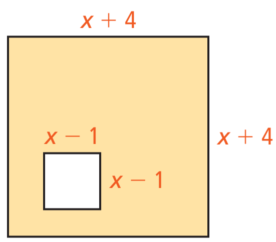
1. $(w+5)^2$ **2.** $(h+2)^2$ **3.** $(3s+9)^2$ **4.** $(2n+7)^2$

5. $(a-8)^2$ **6.** $(k-11)^2$ **7.** $(5m-2)^2$ **8.** $(4x-6)^2$

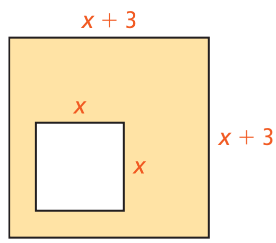
9. Apply Mathematics (1)(A) A company logo is a white square inside a red square. The side length of the white square is $x+2$. The side length of the red square is three times the side length of the white square. What is the area of the red part of the logo? Write your answer in standard form.

Apply Mathematics (1)(A) The figures below are squares. Find an expression for the area of each shaded region. Write your answers in standard form.

10.



11.



12. Explain Mathematical Ideas (1)(G) Describe and correct the error made in simplifying the product.

$\cancel{(3a - 7)^2 = 9a^2 - 21a + 49}$

Simplify each product.

13. $(v^2+6)(v^2-6)$ **14.** $(b+1)(b-1)$ **15.** $(z^2-5)(z^2+5)$

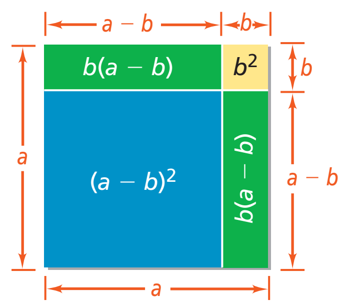
16. $(x-3)(x+3)$ **17.** $(10+y^2)(10-y^2)$ **18.** $(t-13)(t+13)$

19. $(4x+7y)(4x-7y)$ **20.** $(a-6b)(a+6b)$ **21.** $(2g+9h)(2g-9h)$

22. $(r^2+3s)(r^2-3s)$ **23.** $(2p^2+7q)(2p^2-7q)$ **24.** $(3w^3-z^2)(3w^3+z^2)$

25. Apply Mathematics (1)(A) A square deck has a side length of $x+5$. You are expanding the deck so that each side is four times as long as the side length of the original deck. What is the area of the new deck? Write your answer in standard form.

26. Use Representations to Communicate Mathematical Ideas (1)(E) Use the area model at the right to write a second expression for the area of the blue square. Simplify the expression to derive the rule for the square of a binomial of the form $a-b$.



27. Display Mathematical Ideas (1)(G) Give a counterexample to show that $(x+y)^2 = x^2 + y^2$ is false.

28. Does $\left(3\frac{1}{2}\right)^2 = 9\frac{1}{4}$? Justify your answer.

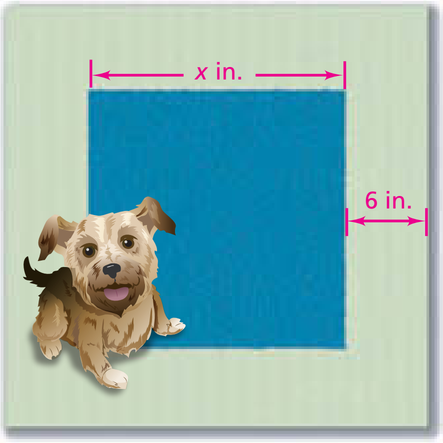
29. The formula $V = \frac{4}{3}\pi r^3$ gives the volume of a sphere with radius r. Find the volume of a sphere with radius $x+3$. Write your answer in standard form.

Simplify each product.

30. $(m + 3n)^2$ **31.** $(2a + b)^2$ **32.** $(4s - t)^2$ **33.** $(g - 7h)^2$

34. $(9k + 2q)^2$ **35.** $(8r - 5s)^2$ **36.** $(p^4 - 9q^2)^2$ **37.** $(a + b + c)^2$

38. **Apply Mathematics (1)(A)** A square green rug has a blue square in the center. The side length of the blue square is x inches. The width of the green band that surrounds the blue square is 6 in. What is the area of the green band?



Select Techniques to Solve Problems (1)(C) **Simplify each product. What technique should you use to solve, mental math or paper and pencil? Explain.**

39. 61^2 **40.** 79^2

41. 302^2 **42.** $42 \cdot 38$

43. $79 \cdot 81$ **44.** $303 \cdot 297$

45. You can use factoring to show that the sum of two multiples of 3 is also a multiple of 3.

If m and n are integers, then $3m$ and $3n$ are multiples of three.

$3m + 3n = 3(m + n)$

Since $m + n$ is an integer, $3(m + n)$ is a multiple of three.

a. Show that if an integer is one more than a multiple of 3, then its square is also one more than a multiple of 3.

b. **Justify Mathematical Arguments (1)(G)** If an integer is two more than at multiple of 3, is its square also two more than a multiple of 3? Explain.

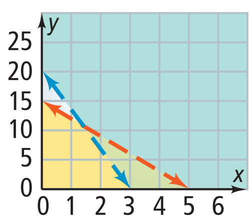
TEXAS End-of-Course PRACTICE

46. Simplify the product $(2x + 5)(2x - 5)$.

A. $4x^2 - 20x - 25$ **B.** $4x^2 + 20x + 25$ **C.** $4x^2 - 25$ **D.** $2x^2 - 5$

47. Sara and Nick sold tickets to a play. Sara sold 20 student tickets and 3 adult tickets for more than \$60. Nick sold 15 student tickets and 5 adult tickets for less than \$75. This information can be represented by $20x + 3y > 60$ and $15x + 5y < 75$, where x is the price of a student ticket and y is the price of an adult ticket. The inequalities are graphed at the right. Which could be the price of a student ticket?

Ticket Sales



F. \$1 **G.** \$2.75 **H.** \$5.50 **J.** \$6

48. Graph the solutions of the system $5x + 4y \geq 20$ and $5x + 4y \leq 20$.

7-5 Factoring $x^2 + bx + c$

TEKS FOCUS

TEKS (10)(E) Factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

TEKS (1)(B) Use a problem-solving model that incorporates analyzing given information, **formulating** a plan or **strategy**, determining a solution, justifying the solution, and evaluating the problem-solving process and the **reasonableness** of the solution.

Additional TEKS (1)(C), (1)(E)

VOCABULARY

- **Formulate** – create with careful effort and purpose. You can formulate a plan or strategy to solve a problem.
- **Strategy** – a plan or method for solving a problem
- **Reasonableness** – the quality of being within the realm of common sense or sound reasoning. The reasonableness of a solution is whether or not the solution makes sense.

ESSENTIAL UNDERSTANDING

You can write some trinomials of the form $x^2 + bx + c$ as the product of two binomials.

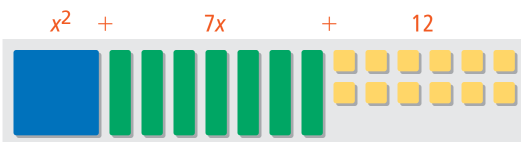
Problem 1

TEKS Process Standard (1)(B)

Using Models to Factor

Write $x^2 + 7x + 12$ as the product of two binomial factors. What tool(s) can you use to help you factor the trinomial?

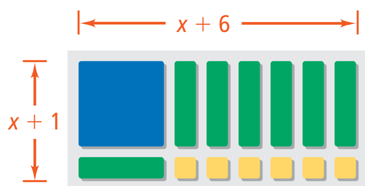
Analyze Given Information Algebra tile manipulatives will help you solve this problem.



Formulate a Strategy To represent the factors of the trinomial, the tiles must be arranged in a rectangle that uses all of the tiles that make up the trinomial.

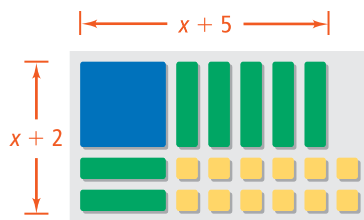
Determine a Solution Use the tiles to form a rectangle.

First try:



There are six  tiles left over.

Second try:



There is one  tile too few.

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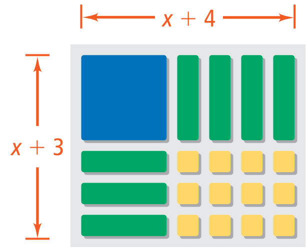
Problem 1 *continued*

Third try:



Correct! There is the exact number of tiles needed.

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Justify the Solution Check the solution by multiplying the factors.

$(x + 3)(x + 4) = x^2 + 4x + 3x + 12$ Multiply binomials.

$= x^2 + 7x + 12$ Combine like terms.

The solution is correct.

Evaluate the Problem-Solving Process Using algebra tiles can be a helpful way to visualize factoring and check your guesses when factoring a trinomial.

Problem 2

TEKS Process Standard (1)(E)

Factoring $x^2 + bx + c$, Where $b > 0$, $c > 0$

What is the factored form of $x^2 + 8x + 15$?

Plan

What is an easy way to organize your factoring?
Use a table to list the pairs of factors of the constant term *c* and the sums of those pairs of factors.

List the pairs of factors of 15. Identify the pair that has a sum of 8.

Factors of 15	Sum of Factors
1 and 15	16
3 and 5	8 ✔

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

Check $(x + 3)(x + 5) = x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$ ✔

Problem 3

TEKS Process Standard (1)(E)

Factoring $x^2 + bx + c$, Where $b < 0$, $c > 0$

What is the factored form of $x^2 - 11x + 24$?

Think

Why look at pairs of *negative* factors of 24?
You want the factors of 24 with a sum of -11. Only two negative numbers have a positive product *and* a negative sum of 11.

List the pairs of negative factors of 24. Identify the pair that has a sum of -11.

Factors of 24	Sum of Factors
–1 and –24	–25
–2 and –12	–14
–3 and –8	–11 ✔
–4 and –6	–10

$$x^2 - 11x + 24 = (x - 3)(x - 8)$$

Check $(x - 3)(x - 8) = x^2 - 8x - 3x + 24$

$= x^2 - 11x + 24$ ✔

Problem 4

Factoring $x^2 + bx + c$, Where $c < 0$

What is the factored form of $x^2 + 2x - 15$?

Think

What's another way to do this problem? Find two positive factors of 15 that *differ* by 2. The factors are 3 and 5. Then attach a negative sign to one of the factors so that their sum is positive. You get -3 and 5.

Identify the pair of factors of -15 that has a sum of 2.

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Factors of -15	Sum of Factors
1 and -15	-14
-1 and 15	14
3 and -5	-2
-3 and 5	2 ✔

Problem 5

Applying Factoring Trinomials

Geometry **The area of a rectangle is given by the trinomial $x^2 - 2x - 35$. What are the possible dimensions of the rectangle? Use factoring.**

Know
The area of the rectangle

Need
Possible dimensions of the rectangle

Plan
Area = length × width, so factor the trinomial for area as the product of binomials that represent the length and width.

To factor $x^2 - 2x - 35$, identify the pair of factors of -35 that has a sum of -2.

$x^2 - 2x - 35 = (x + 5)(x - 7)$

So the possible dimensions of the rectangle are $x + 5$ and $x - 7$.

Factors of -35	Sum of Factors
1 and -35	-34
-1 and 35	34
5 and -7	-2 ✔
-5 and 7	2

Problem 6

Factoring a Trinomial With Two Variables

What is the factored form of $x^2 + 6xy - 55y^2$?

Plan

Is this problem similar to one you've seen before? Yes. This problem is similar to factoring a trinomial in one variable of the form $x^2 + bx + c$, where $c < 0$.

List the pairs of factors of -55. Identify the pair that has a sum of 6.

$x^2 + 6xy - 55y^2 = (x - 5y)(x + 11y)$

Factors of -55	Sum of Factors
1 and -55	-54
-1 and 55	54
5 and -11	-6
-5 and 11	6 ✔

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Select Tools to Solve Problems (1)(C) Factor the trinomial. Explain what tool or technique you used.

1. $x^2 + 9x + 8$ **2.** $x^2 + 8x + 12$ **3.** $x^2 + 5x + 6$

4. **Use a Problem-Solving Model (1)(B)** Write $x^2 + 7x + 10$ as a product of binomials. Use a problem-solving model by analyzing the given information, formulating a plan, determining a solution, justifying the solution, and evaluating the problem-solving process.

Complete.

5. $t^2 - 10t + 24 = (t - 4)(t - \blacksquare)$ **6.** $v^2 + 12v + 20 = (v + 10)(v + \blacksquare)$

7. $q^2 + 3q - 54 = (q - 6)(q + \blacksquare)$ **8.** $z^2 - 2z - 48 = (z - 8)(z + \blacksquare)$

9. **Explain Mathematical Ideas (1)(G)** Suppose you can factor $x^2 + bx + c$ as $(x + p)(x + q)$.

a. Explain what you know about p and q when $c > 0$.

b. Explain what you know about p and q when $c < 0$.

Factor each expression. Check your answer.

10. $n^2 - 15n + 56$ **11.** $r^2 - 11r + 24$ **12.** $q^2 - 8q + 12$

13. $x^2 + 5x - 6$ **14.** $v^2 + 5v - 36$ **15.** $n^2 - 3n - 10$

16. **Apply Mathematics (1)(A)** The area of a rectangular desk is given by the trinomial $d^2 - 7d - 18$. What are the possible dimensions of the desk? Use factoring.

17. **Apply Mathematics (1)(A)** The area of a rectangular rug is given by the trinomial $r^2 - 3r - 4$. What are the possible dimensions of the rug? Use factoring.

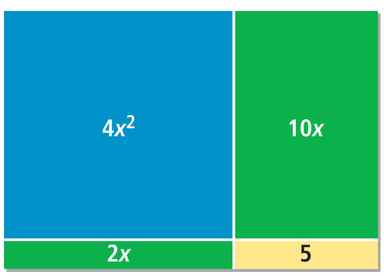
Factor each expression.

18. $r^2 + 19rs + 90s^2$ **19.** $g^2 - 12gh + 35h^2$ **20.** $m^2 - 3mn - 28n^2$

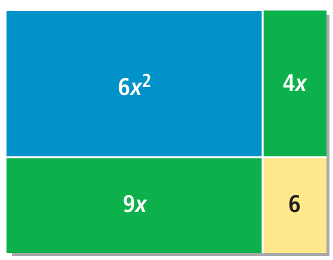
21. $x^2 + 3xy - 18y^2$ **22.** $w^2 - 14wz + 40z^2$ **23.** $p^2 + 11pq + 24q^2$

Use Representations to Communicate Mathematical Ideas (1)(E) Write the standard form of each polynomial modeled below. Then factor each expression.

24.



25.



26. **Explain Mathematical Ideas (1)(G)** Describe and correct the error made in factoring the trinomial.

$$\cancel{x^2 - 10x - 24 = (x - 6)(x - 4)}$$

27. The area of a parallelogram is given by the trinomial $x^2 - 14x + 24$. The base of the parallelogram is $x - 2$. What is an expression for the height of the parallelogram?

28. **Apply Mathematics (1)(A)** A rectangular skateboard park has an area of $x^2 + 15x + 54$. What are the possible dimensions of the park? Use factoring.

Factor each expression.

29. $x^2 + 27x + 50$
30. $g^2 - 18g + 45$
31. $k^2 - 18k - 63$
32. $d^2 + 30d - 64$
33. $s^2 - 10st - 75t^2$
34. $h^2 + 9hj - 90j^2$

35. **Explain Mathematical Ideas (1)(G)** Let $x^2 - 13x - 30 = (x + p)(x + q)$.
    a. What do you know about the signs of p and q?
    b. Suppose $|p| > |q|$. Which number, p or q, is a negative integer? Explain.

36. **Explain Mathematical Ideas (1)(G)** Let $x^2 + 13x - 30 = (x + p)(x + q)$.
    a. What do you know about the signs of p and q?
    b. Suppose $|p| > |q|$. Which number, p or q, is a negative integer? Explain.

Factor each trinomial.

37. $x^{12} + 12x^6 + 35$
38. $t^8 + 5t^4 - 24$
39. $r^6 - 21r^3 + 80$
40. $m^{10} + 18m^5 + 17$
41. $x^{12} - 19x^6 - 120$
42. $p^6 + 14p^3 - 72$

TEXAS End-of-Course PRACTICE

43. What is the factored form of $x^2 + x - 42$?
    A. $(x - 7)(x - 6)$ **B.** $(x - 7)(x + 6)$ **C.** $(x + 7)(x - 6)$ **D.** $(x + 7)(x + 6)$

44. What is the solution of the equation $6x + 7 = 25$?
    F. 2 **G.** 3 **H.** $5\frac{1}{3}$ **J.** 8

45. A museum charges an admission price of \$12 per person when you buy tickets online. There is also a \$5 charge per order. You spend \$65 purchasing p tickets online. Which equation best represents this situation?
    A. $12p + 5 = 65$ **B.** $5p + 12 = 65$ **C.** $12p - 5 = 65$ **D.** $65p + 12 = 5$

7-6 Factoring $ax^2 + bx + c$

TEKS FOCUS

TEKS (10)(E) Factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(C), (10)(D)

VOCABULARY

- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

You can write some trinomials of the form $ax^2 + bx + c$ as the product of two binomials.

Problem 1

TEKS Process Standard (1)(E)

Factoring When *ac* Is Positive

What is the factored form of $5x^2 + 11x + 2$?

Think

Will the process still work if you write $5x^2 + 10x + x + 2$?
Yes. You can rewrite this alternate expression as $5x(x + 2) + (x + 2)$, which equals $(5x + 1)(x + 2)$.

Step 1 Find factors of ac that have sum b. Since $ac = 10$ and $b = 11$, find positive factors of 10 that have sum 11.

Factors of 10	1, 10	2, 5
Sum of Factors	11 ✔	7

Step 2 To factor the trinomial, use the factors you found to rewrite bx.

$5x^2 + 11x + 2 = 5x^2 + 1x + 10x + 2$ Rewrite bx: $11x = 1x + 10x$.

$= x(5x + 1) + 2(5x + 1)$ Factor out the GCF of each pair of terms.

$= (x + 2)(5x + 1)$ Distributive Property

Problem 2

Factoring When *ac* Is Negative

What is the factored form of $3x^2 + 4x - 15$?

Plan

Can you apply the steps for Problem 1 to this problem?
Yes. Your goal is still to find factors of ac that have sum b. Because $ac < 0$, the factors must have different signs.

Step 1 Find factors of ac that have sum b. Since $ac = -45$ and $b = 4$, find factors of -45 that have sum 4.

Factors of −45	1, −45	−1, 45	3, −15	−3, 15	5, −9	−5, 9
Sum of Factors	−44	44	−12	12	−4	4 ✔

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Problem 2 *continued*

Step 2 To factor the trinomial, use the factors you found to rewrite bx.

$3x^2 + 4x - 15 = 3x^2 - 5x + 9x - 15$ Rewrite bx: $4x = -5x + 9x$.

$= x(3x - 5) + 3(3x - 5)$ Factor out the GCF of each pair of terms.

$= (3x - 5)(x + 3)$ Distributive Property

Problem 3

Applying Trinomial Factoring

Plan

How can you find the dimensions of the rectangle?
Factor the rectangle's area as the product of two binomials, one of which is the width. The other must be the length since area = length · width.

Geometry **The area of a rectangle is $2x^2 - 13x - 7$. What are the possible dimensions of the rectangle? Use factoring.**

Step 1 Find factors of ac that have sum b. Since $ac = -14$ and $b = -13$, find factors of -14 that have sum -13.

Factors of −14	1, −14	−1, 14	2, −7	−2, 7
Sum of Factors	−13 ✔	13	−5	5

Step 2 To factor the trinomial, use the factors you found to rewrite bx.

$2x^2 - 13x - 7 = 2x^2 + x - 14x - 7$ Rewrite bx: $-13x = x - 14x$.

$= x(2x + 1) - 7(2x + 1)$ Factor out the GCF of each pair of terms.

$= (2x + 1)(x - 7)$ Distributive Property

The possible dimensions of the rectangle are $2x + 1$ and $x - 7$.

Problem 4

TEKS Process Standard (1)(E)

Factoring Out a Monomial First

What is the factored form of $18x^2 - 33x + 12$?

Plan

How can you simplify this problem?
Factor out the GCF of the trinomial's terms. The trinomial that remains is similar to those in Problems 1–3.

Think

Factor out the GCF.

Write

$18x^2 - 33x + 12 = 3(6x^2 - 11x + 4)$

Think

Factor $6x^2 - 11x + 4$. Since $ac = 24$ and $b = -11$, find negative factors of 24 that have sum -11.

Write

Factors of 24	−1, −24	−2, −12	−3, −8	−4, −6
Sum of Factors	−25	−14	−11 ✔	−10

Think

Rewrite the term bx. Then use the Distributive Property to finish factoring.

Write

$3(6x^2 - 3x - 8x + 4)$
$3[3x(2x - 1) - 4(2x - 1)]$
$3(3x - 4)(2x - 1)$

PRACTICE and APPLICATION EXERCISES

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Factor each expression.

1. $2x^2 + 13x + 6$
2. $3d^2 + 23d + 14$
3. $4n^2 - 8n + 3$
4. $4p^2 + 7p + 3$
5. $6r^2 - 23r + 20$
6. $8g^2 - 14g + 3$
7. $3x^2 + 23x - 36$
8. $4w^2 - 5w - 6$
9. $4d^2 - 4d - 35$

10. **Apply Mathematics (1)(A)** The area of a rectangular kitchen tile is $8x^2 + 30x + 7$. What are the possible dimensions of the tile? Use factoring.

11. **Apply Mathematics (1)(A)** The area of a rectangular knitted blanket is $15x^2 - 14x - 8$. What are the possible dimensions of the blanket? Use factoring.

Factor each expression completely.

12. $12p^2 + 20p - 8$
13. $8v^2 + 34v - 30$
14. $6s^2 + 57s + 72$
15. $20w^2 - 45w + 10$
16. $12x^2 - 46x - 8$
17. $9r^2 + 3r - 30$

Select Techniques to Solve Problems (1)(C) **Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.**

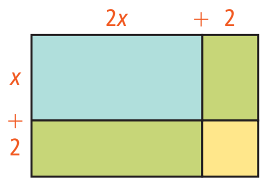
18. $4s^2 + \blacksquare s + 10$
19. $15v^2 + \blacksquare v - 24$
20. $35m^2 + \blacksquare m - 16$
21. $9g^2 + \blacksquare g + 4$
22. $6n^2 + \blacksquare n + 28$
23. $8r^2 + \blacksquare r - 42$

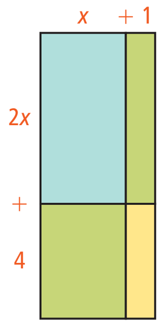
24. **Explain Mathematical Ideas (1)(G)** Describe and correct the error made in factoring the expression at the right.

$$3x^2 - 16x - 12 = 3x^2 + 4x - 20x - 12$$
$$= x(3x + 4) - 4(5x + 3)$$
$$= (x - 4)(3x + 4)(5x + 3)$$

25. A triangle has an area of $9x^2 - 9x - 10$. The base of the triangle is $3x - 5$. What is the height of the triangle?

26. **a.** Write each area as a product of two binomials.





b. Are the products equal?

c. **Explain Mathematical Ideas (1)(G)** Explain how the two products you found in part (a) can equal the same trinomial.

27. **Apply Mathematics (1)(A)** The top of a rectangular table has an area of $18x^2 + 69x + 60$. The width of the table is $3x + 4$. What is the length of the table?

Factor each expression completely.

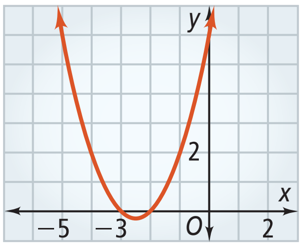
28. $28h^2 + 28h - 56$
29. $21y^2 + 72y - 48$
30. $55n^2 - 52n + 12$
31. $36p^2 + 114p - 20$
32. $63g^2 - 89g + 30$
33. $99v^2 - 92v + 9$

34. **Justify Mathematical Arguments (1)(G)** If a and c in $ax^2 + bx + c$ are prime numbers and the trinomial is factorable, how many positive values are possible for b? Explain your reasoning.

Factor each expression completely.

35. $56x^3 + 43x^2 + 5x$
36. $49p^2 + 63pq - 36q^2$
37. $108g^2h - 162gh + 54h$

38. The graph of the function $y = x^2 + 5x + 6$ is shown below.



a. What are the x-intercepts?

b. Factor $x^2 + 5x + 6$.

c. **Explain Mathematical Ideas (1)(G)** Describe the relationship between the binomial factors you found in part (b) and the x-intercepts.

TEXAS End-of-Course PRACTICE

39. What is the missing value in the statement $7x^2 - 61x - 18 = (7x + 2)(x - \blacksquare)$?
40. What is the y-intercept of the graph of $-3x + y = 1$?
41. A book has a spine 4.3×10^{-2} m thick. What is 4.3×10^{-2} written in standard form?
42. The number of tourists who visit a certain country is expected to be 440% greater in the year 2020 than in the year 2000. What is 440% written as a decimal?

7-7 Factoring Special Cases

TEKS FOCUS

TEKS (10)(F) Decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(A), (10)(E)

VOCABULARY

- **Difference of two squares** – A difference of two squares is an expression of the form $a^2 - b^2$. It can be factored as $(a + b)(a - b)$.
- **Perfect-square trinomial** – any trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

ESSENTIAL UNDERSTANDING

You can factor some trinomials by "reversing" the rules for multiplying special case binomials that you learned in Lesson 7-4.

take note

Key Concept Factoring Perfect-Square Trinomials

Algebra For every real number a and b:

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

Examples $x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2$

$4n^2 - 12n + 9 = (2n - 3)(2n - 3) = (2n - 3)^2$

take note

Key Concept Factoring a Difference of Two Squares

Algebra For all real numbers a and b:

$$a^2 - b^2 = (a + b)(a - b)$$

Examples $x^2 - 64 = (x + 8)(x - 8)$

$25x^2 - 36 = (5x + 6)(5x - 6)$

Problem 1

Factoring a Perfect-Square Trinomial

Think

Will the answer have the form $(a + b)^2$ or $(a - b)^2$?
The middle term $-12x$ has a negative coefficient, so the factored expression will have the form $(a - b)^2$.

What is the factored form of $x^2 - 12x + 36$?

$x^2 - 12x + 36 = x^2 - 12x + 6^2$ — Write the last term as a square.

$= x^2 - 2(x)(6) + 6^2$ — Does middle term equal $-2ab$? $-12x = -2(x)(6)$ ✔

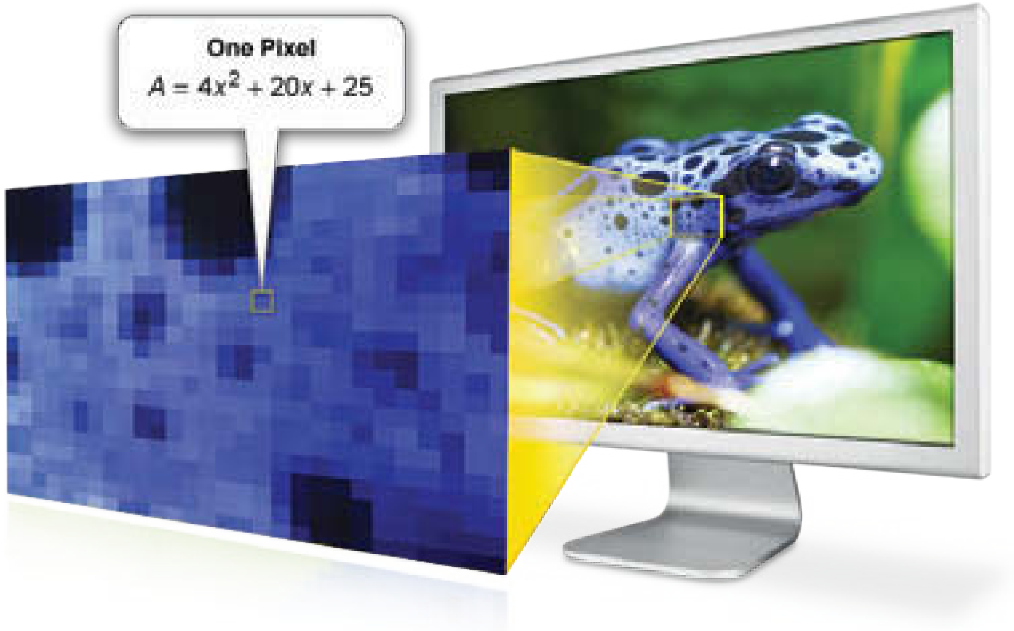
$= (x - 6)^2$ — Write as the square of a binomial.

Problem 2

TEKS Process Standard (1)(A)

Factoring to Find a Length STEM

Computers **Digital images are composed of thousands of tiny pixels rendered as squares, as shown below. Suppose the area of a pixel is $4x^2 + 20x + 25$. What is the length of one side of the pixel?**



Plan

How can you find the side length?
Since the pixel's area is its side length squared, factor the expression for area as the square of a binomial. The binomial is the side length.

$4x^2 + 20x + 25 = (2x)^2 + 20x + 5^2$ — Write first and last terms as squares.

$= (2x)^2 + 2(2x)(5) + 5^2$ — Does middle term equal $2ab$? $20x = 2(2x)(5)$ ✔

$= (2x + 5)^2$ — Write as the square of a binomial.

The length of one side of the pixel is $2x + 5$.

Problem 3

TEKS Process Standard (1)(G)

Factoring a Difference of Two Squares

What is the factored form of $z^2 - 9$?

Plan

Can you use the rule for the difference of two squares?
Yes. The binomial is a difference *and* both its terms are perfect squares.

Think	Write
Rewrite 9 as a square.	$z^2 - 9 = z^2 - 3^2$
Factor using the rule for a difference of two squares.	$= (z + 3)(z - 3)$
Check your answer by multiplying the factored form.	$(z + 3)(z - 3) = z^2 - 3z + 3z - 9$ $= z^2 - 9$ ✔

Problem 4

Think

When is a term of the form ax^2 a perfect square?
ax^2 is a perfect square when a is a perfect square. For example, $16x^2$ is a perfect square but $17x^2$ is not.

Factoring a Difference of Two Squares

What is the factored form of $16x^2 - 81$?

$16x^2 - 81 = (4x)^2 - 9^2$ Write each term as a square.

$= (4x + 9)(4x - 9)$ Use the rule for the difference of two squares.

Problem 5

Think

Is $24g^2 - 6$ a difference of two squares?
No. $24g^2$ and 6 are not perfect squares. To get a difference of squares, you must first factor out the GCF.

Factoring Out a Common Factor

What is the factored form of $24g^2 - 6$?

$24g^2 - 6 = 6(4g^2 - 1)$ Factor out the GCF, 6.

$= 6[(2g)^2 - 1^2]$ Write the difference as $a^2 - b^2$.

$= 6(2g + 1)(2g - 1)$ Use the rule for the difference of two squares.

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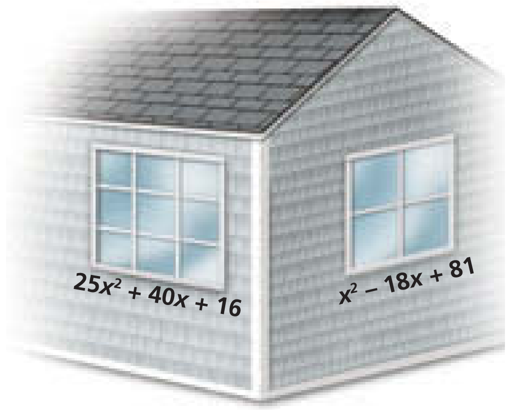
Factor each expression.

1. $h^2 + 8h + 16$ **2.** $v^2 - 10v + 25$ **3.** $d^2 - 20d + 100$

4. $36s^2 - 60s + 25$ **5.** $25z^2 + 40z + 16$ **6.** $49g^2 - 84g + 36$

7. Apply Mathematics (1)(A) Two square windows and their areas are shown at the right. What is an expression that represents the difference of the areas of the windows? Show two different ways to find the solution.

8. Apply Mathematics (1)(A) A square rug has an area of $49x^2 - 56x + 16$. A second square rug has an area of $16x^2 + 24x + 9$. What is an expression that represents the difference of the areas of the rugs? Show two different ways to find the solution.



Factor each expression completely.

9. $t^2 - 25$ **10.** $k^2 - 64$ **11.** $m^2 - 225$

12. $64q^2 - 81$ **13.** $16x^2 - 121$ **14.** $9n^2 - 400$

15. $2h^2 - 2$ **16.** $27w^2 - 12$ **17.** $80g^2 - 45$

18. Rewrite the expression $x^4 - y^4$ so that it is a difference of two squares. Then factor the expression completely.

19. Explain Mathematical Ideas (1)(G) Describe and correct the error made in factoring.

$\cancel{9x^2 - 49 = (9x + 7)(9x - 7)}$

20. Explain Mathematical Ideas (1)(G) Summarize the procedure for factoring a difference of two squares. Give at least two examples.

For Exercises 21–25, find a pair of factors for each number by using the difference of two squares.

Sample	$117 = 121 - 4$	Write 117 as the difference of two squares.
	$= 11^2 - 2^2$	Write each term as a square.
	$= (11 + 2)(11 - 2)$	Use the rule for the difference of squares.
	$= (13)(9)$	Simplify.

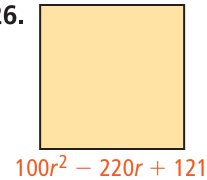
21. 143 **22.** 99 **23.** 224 **24.** 84

25. a. Display Mathematical Ideas (1)(G) Write an expression that is a perfect-square trinomial.

b. Explain how you know your trinomial is a perfect-square trinomial.

The given expression represents the area. Find the side length of each square.

26.



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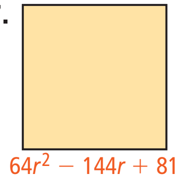

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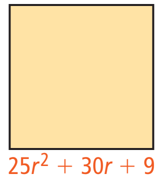


29. a. Factor $4x^2 - 100$ by removing the common monomial factor and then using the difference-of-squares rule to factor the remaining expression.

b. Factor $4x^2 - 100$ by using the difference-of-squares rule and removing the common monomial factors.

c. Explain Mathematical Ideas (1)(G) Why can you factor $4x^2 - 100$ in two different ways?

d. Can you factor $3x^2 - 75$ in the two ways you factored $4x^2 - 100$ in parts (a) and (b)? Explain your answer.

Factor each expression completely.

30. $64r^6 - 144r^3 + 81$

31. $p^6 + 40p^3q + 400q^2$

32. $36m^4 + 84m^2 + 49$

33. $108n^6 - 147$

34. $x^{20} - 4x^{10}y^5 + 4y^{10}$

35. $256g^4 - 100h^6$

36. The binomial $16 - 81n^4$ can be factored twice using the difference-of-squares rule.

a. Factor $16 - 81n^4$ completely.

b. Analyze Mathematical Relationships (1)(F) What characteristics do 16 and $81n^4$ share that make this possible?

c. Display Mathematical Ideas (1)(G) Write another binomial that can be factored twice using the difference-of-squares rule.

TEXAS End-of-Course PRACTICE

37. What is the factored form of $4x^2 - 20x + 25$?

A. $(2x + 5)(2x - 5)$

B. $(2x - 5)(2x - 5)$

C. $(4x - 5)(4x - 5)$

D. $(4x + 5)(4x - 5)$

38. Which equation has -2 as its solution?

F. $x + 3 = 2x + 1$

G. $x - 5 = 2x - 7$

H. $2x + 5 = 5x + 11$

J. $3x + 1 = x - 5$

39. A film club sponsors a film fest at a local movie theater. Renting the theater costs \$190. The admission is \$2 per person. Write and graph an equation that relates the film club's total cost c and the number of people p who attend the film fest.

7-8 Factoring by Grouping

TEKS FOCUS

TEKS (10)(B) Multiply polynomials of degree one and degree two.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(A)

VOCABULARY

- **Factoring by grouping** – a method of factoring that uses the Distributive Property to remove a common binomial factor of two pairs of terms
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

Some polynomials of a degree greater than 2 can be factored.

take note

Summary Factoring Polynomials

1. Factor out the greatest common factor (GCF).
2. If the polynomial has two terms or three terms, look for a difference of two squares, a perfect-square trinomial, or a pair of binomial factors.
3. If the polynomial has four or more terms, group terms and factor to find common binomial factors.
4. As a final check, make sure there are no common factors other than 1.

Problem 1

Factoring a Cubic Polynomial

Plan

How should you group the terms of the polynomial?
First, group the two terms with the highest degrees. If that doesn't work, try another grouping. Your goal is to find a common binomial factor.

What is the factored form of $3n^3 - 12n^2 + 2n - 8$?

$3n^3 - 12n^2 + 2n - 8 = 3n^2(n - 4) + 2(n - 4)$ Factor out the GCF of each group of two terms.

$= (3n^2 + 2)(n - 4)$ Factor out the common factor, $n - 4$.

Check $(3n^2 + 2)(n - 4) = 3n^3 - 12n^2 + 2n - 8$ ✔

Problem 2

Think

Do the terms share any numerical or variable factors?
Yes. The terms have a common numerical factor of 4 and a common variable factor of q.
The GCF is $4q$.

Factoring a Polynomial Completely

What is the factored form of $4q^4 - 8q^3 + 12q^2 - 24q$? Factor completely.

$$4q^4 - 8q^3 + 12q^2 - 24q = 4q\left(q^3 - 2q^2 + 3q - 6\right) \quad \text{Factor out the GCF.}$$

$$= 4q\left[q^2(q-2) + 3(q-2)\right] \quad \text{Factor by grouping.}$$

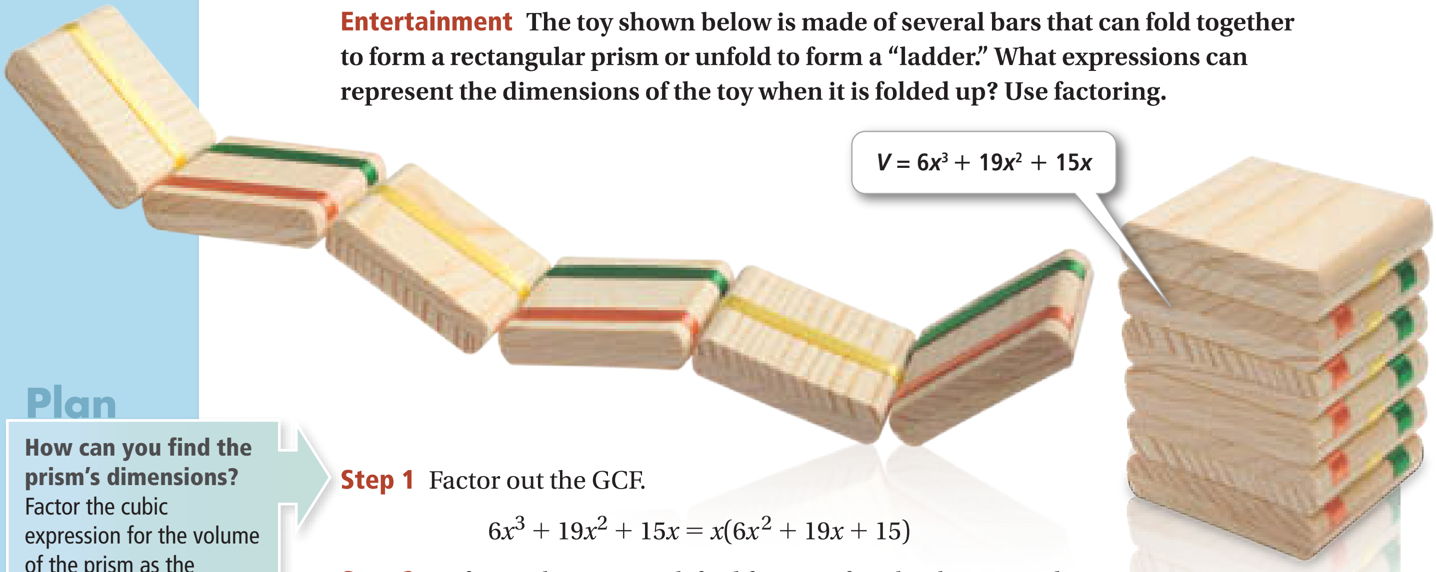
$$= 4q\left(q^2 + 3\right)(q-2) \quad \text{Factor again.}$$

Problem 3

TEKS Process Standard (1)(E)

Finding the Dimensions of a Rectangular Prism

Entertainment **The toy shown below is made of several bars that can fold together to form a rectangular prism or unfold to form a "ladder." What expressions can represent the dimensions of the toy when it is folded up? Use factoring.**



Plan

How can you find the prism's dimensions?
Factor the cubic expression for the volume of the prism as the product of three linear expressions. Each linear expression represents a dimension.

Step 1 Factor out the GCF.

$$6x^3 + 19x^2 + 15x = x(6x^2 + 19x + 15)$$

Step 2 To factor the trinomial, find factors of ac that have sum b. Since $ac = 90$ and $b = 19$, find factors of 90 that have sum 19.

Factors of 90	1, 90	2, 45	3, 30	5, 18	6, 15	9, 10
Sum of Factors	91	47	33	23	21	19 ✔

Step 3 To factor the trinomial, use the factors you found to rewrite bx.

$$x(6x^2 + 19x + 15) = x(6x^2 + 9x + 10x + 15) \quad \text{Rewrite } bx\text{: } 19x = 9x + 10x.$$

$$= x[3x(2x + 3) + 5(2x + 3)] \quad \text{Factor by grouping.}$$

$$= x(3x + 5)(2x + 3) \quad \text{Distributive Property}$$

The possible dimensions are x, $3x + 5$, and $2x + 3$.

PRACTICE and APPLICATION EXERCISES

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Find the GCF of the first two terms and the GCF of the last two terms for each polynomial.

1. $2z^3 + 6z^2 + 3z + 9$ **2.** $10g^3 - 25g^2 + 4g - 10$ **3.** $2r^3 + 12r^2 - 5r - 30$

Factor each expression.

4. $15q^3 + 40q^2 + 3q + 8$

5. $14y^3 + 8y^2 + 7y + 4$

6. $14z^3 - 35z^2 + 16z - 40$

7. $20v^3 + 24v^2 - 25v - 30$

8. $18h^3 + 45h^2 - 8h - 20$

9. $12y^3 + 4y^2 - 9y - 3$

10. Analyze Mathematical Relationships (1)(F) Describe and correct the error made in factoring completely.

$$
\begin{aligned}
4x^4 + 12x^3 + 8x^2 + 24x &= 4(x^4 + 3x^3 + 2x^2 + 6x) \\
&= 4[x^3(x+3) + 2x(x+3)] \\
&= 4(x^3 + 2x)(x+3)
\end{aligned}
$$

11. a. Factor $(20x^3 - 5x^2) + (44x - 11)$.

b. Factor $(20x^3 + 44x) + (-5x^2 - 11)$.

c. Explain Mathematical Ideas (1)(G) Why can you factor the same polynomial using different pairs of terms?

12. Explain Mathematical Ideas (1)(G) Describe how to factor the expression $6x^5 + 4x^4 + 12x^3 + 8x^2 + 9x + 6$.

Factor completely.

13. $8p^3 - 32p^2 + 28p - 112$

14. $3w^4 - 2w^3 + 18w^2 - 12w$

15. $5g^4 - 5g^3 + 20g^2 - 20g$

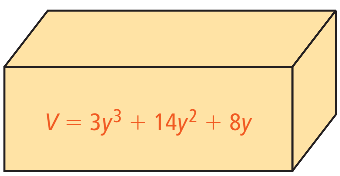
16. $6q^4 + 3q^3 - 24q^2 - 12q$

17. $36v^3 - 126v^2 + 48v - 168$

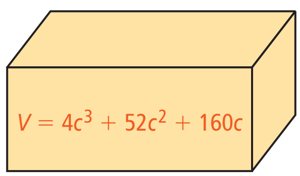
18. $4d^3 - 6d^2 + 16d - 24$

Create Representations to Communicate Mathematical Ideas (1)(E) Find expressions for the possible dimensions of each rectangular prism.

19.



20.



21. Apply Mathematics (1)(A) A trunk in the shape of a rectangular prism has a volume of $6x^3 + 38x^2 - 28x$. What expressions can represent the dimensions of the trunk?

Factor completely.

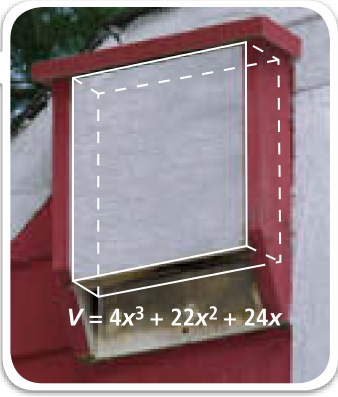
22. $60y^4 - 300y^3 - 42y^2 + 210y$

23. $8m^3 + 32m^2 + 40m + 160$

24. $10p^2 - 5pq - 180q^2$

25. $9t^3 - 90t^2 + 144t$

26. **Apply Mathematics (1)(A)** Bat houses, such as the one at the right, are large wooden structures that people mount on buildings to attract bats. What expressions can represent the dimensions of the bat house?



27. **Display Mathematical Ideas (1)(G)** Write a four-term polynomial that you can factor by grouping. Factor your polynomial.

28. **Apply Mathematics (1)(A)** The pedestal of a sculpture is a rectangular prism with a volume of $63x^3 - 28x$. What expressions can represent the dimensions of the pedestal? Use factoring.

Factor by grouping.

29. $p^2m + p^2n^5 + qm + qn^5$

30. $30g^5 + 24g^3h - 35g^2h^2 - 28h^3$

31. **Apply Mathematics (1)(A)** The polynomial $2\pi x^3 + 12\pi x^2 + 18\pi x$ represents the volume of a cylinder. The formula for the volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.

a. Factor $2\pi x^3 + 12\pi x^2 + 18\pi x$.

b. Based on your answer to part (a), write an expression for a possible radius of the cylinder.

You can write the number 63 as $2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$. For Exercises 32 and 33, factor each expression by grouping. Then simplify the powers of 2 to write 63 as the product of two numbers.

32. $\left(2^5 + 2^4 + 2^3\right) + \left(2^2 + 2^1 + 2^0\right)$

33. $\left(2^5 + 2^4\right) + \left(2^3 + 2^2\right) + \left(2^1 + 2^0\right)$

TEXAS End-of-Course PRACTICE

34. What is $30z^3 - 12z^2 + 120z - 48$ factored completely?

A. $2(15z^3 - 6z^2 + 60z - 24)$

B. $(6z^2 + 24)(5z - 2)$

C. $6(5z^3 - 2z^2 + 20z - 8)$

D. $6(z^2 + 4)(5z - 2)$

35. What is the solution of the inequality $7 < -2x + 5$?

F. $x > -1$ **G.** $x < -1$ **H.** $x > 1$ **J.** $x < 1$

36. Factor $10r^4 + 30r^3 + 5r^2 + 15r$ completely. Show your work.

7-9 Simplifying Rational Expressions

TEKS FOCUS

TEKS (10)(D) Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(A), (10)(E), (10)(F)

VOCABULARY

- **Excluded value** – a value of a variable for which a rational expression is undefined
- **Rational expression** – a ratio of two polynomials where the value of the variable cannot make the denominator equal to 0

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

ESSENTIAL UNDERSTANDING

The simplified form of a rational expression is like the simplified form of a numerical fraction. The numerator and denominator have no common factor other than 1. To simplify a rational expression, divide out common factors from the numerator and denominator.

take note Key Concept Rational Expressions

An expression of the form $\frac{\text{polynomial}}{\text{polynomial}}$ is a **rational expression**.

Like a numerical fraction, a rational expression is undefined when the denominator is 0. A value of a variable for which a rational expression is undefined is an **excluded value**.

Problem 1

Simplifying a Rational Expression

What is the simplified form of $\frac{x-1}{5x-5}$? State any excluded values.

$\frac{x-1}{5x-5} = \frac{x-1}{5(x-1)}$ Factor the denominator. The numerator cannot be factored.

$= \frac{\cancel{x-1}^{1}}{5_{1}\cancel{(x-1)}}$ Divide out the common factor, $x - 1$.

$= \frac{1}{5}$ Simplify.

The denominator of the original expression is 0 when $x = 1$. The simplified form is $\frac{1}{5}$, where $x \neq 1$.

Think

Should you use the simplified form to find excluded values?
No. You must check the original expression to see which values of x make the denominator 0.

Problem 2

TEKS Process Standard (1)(G)

Simplifying a Rational Expression Containing a Trinomial

What is the simplified form of $\frac{3x - 6}{x^2 + x - 6}$? State any excluded values.

Think

To see if there are any common factors, factor the numerator and the denominator.

Divide out the common factor, $x - 2$. Simplify.

State the simplified form with any restrictions on the variable.

Write

$$\frac{3x-6}{x^2+x-6} = \frac{3(x-2)}{(x+3)(x-2)}$$

$$= \frac{3\cancel{(x-2)}^1}{(x+3)_1\cancel{(x-2)}}$$

$$= \frac{3}{(x+3)}$$

The denominator of the original expression is 0 when $x = -3$ or $x = 2$. So the simplified form is $\frac{3}{x+3}$, where $x \neq -3$ and $x \neq 2$.

Think

Could you also find the restricted values *before* simplifying?
Yes. You use the original expression to find the restrictions on x, so you don't need to simplify the expression first.

Problem 3

Recognizing Opposite Factors

What is the simplified form of $\frac{4 - x^2}{7x - 14}$? State any excluded values.

$$\frac{4-x^2}{7x-14} = \frac{(2-x)(2+x)}{7(x-2)}$$ Factor the numerator and the denominator.

$$= \frac{-1(x-2)(2+x)}{7(x-2)}$$ Factor -1 from $2 - x$.

$$= \frac{-1\cancel{(x-2)}^1(2+x)}{7_1\cancel{(x-2)}}$$ Divide out the common factor, $x - 2$.

$$= -\frac{x+2}{7}$$ Simplify.

The denominator of the original expression is 0 when $x = 2$. The simplified form is $-\frac{x+2}{7}$, where $x \neq 2$.

Plan

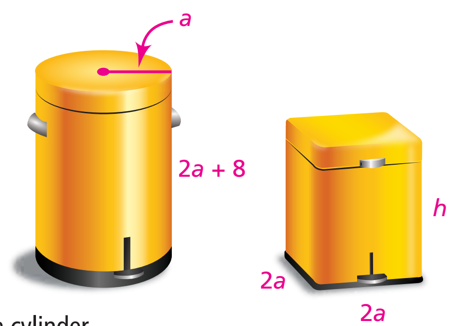
When should you factor -1 from an expression?
You should factor -1 from $a - x$ when factoring -1 results in a common factor.

Problem 4

TEKS Process Standard (1)(A)

Using a Rational Expression

Shopping **You are choosing between the two wastebaskets that have the shape of the figures at the right. They both have the same volume. What is the height *h* of the rectangular wastebasket? Give your answer in terms of *a*.**



Plan

Is there another way to solve the problem?
Yes. You can set the volumes equal to each other and then solve for *h*.

Step 1 Find the volume of the cylinder.

$V = \pi r^2 h$	Formula for volume of a cylinder
$= \pi a^2(2a + 8)$	Substitute a for r and $2a + 8$ for h.

Step 2 Find the height of a rectangular prism with volume $\pi a^2(2a + 8)$ and base area $B = (2a)^2 = 4a^2$.

$V = Bh$	Formula for volume of a prism
$h = \frac{V}{B}$	Solve for h.
$= \frac{\pi a^2(2a + 8)}{4a^2}$	Substitute the volume of the cylinder for the volume of the rectangular prism and $4a^2$ for B.
$= \frac{\pi a^2(2)(a + 4)}{4a^2}$	Factor.
$= \frac{\pi \overset{1}{\cancel{a^2}}\overset{1}{\cancel{(2)}}(a + 4)}{\underset{2}{\cancel{4}}\underset{1}{\cancel{a^2}}}$	Divide out common factors, 2 and a^2.
$= \frac{\pi(a + 4)}{2}$	Simplify.

The height of the rectangular prism is $\frac{\pi(a + 4)}{2}$.

PRACTICE and APPLICATION EXERCISES

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Simplify each expression. State any excluded values.

1. $\frac{6a + 9}{12}$ **2.** $\frac{4x^3}{28x^4}$ **3.** $\frac{2m - 5}{6m - 15}$

4. $\frac{2x^2 + 2x}{3x^2 + 3x}$ **5.** $\frac{2b - 8}{b^2 - 16}$ **6.** $\frac{m + 6}{m^2 - m - 42}$

7. $\frac{c^2 - 6c + 8}{c^2 + c - 6}$ **8.** $\frac{b^2 + 8b + 15}{b + 5}$ **9.** $\frac{m + 4}{m^2 + 2m - 8}$

10. $\frac{m - 2}{4 - 2m}$ **11.** $\frac{v - 5}{25 - v^2}$ **12.** $\frac{4 - w}{w^2 - 8w + 16}$

13. The length of a rectangular prism is 5 more than twice the width w. The volume of the prism is $2w^3 + 7w^2 + 5w$. What is a simplified expression for the height of the prism?

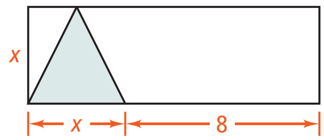
14. Rectangle A has length $2x + 6$ and width $3x$. Rectangle B has length $x + 2$ and an area 12 square units greater than Rectangle A's area. What is a simplified expression for the width of Rectangle B?

Simplify each expression. State any excluded values.

15. $\frac{7z^2 + 23z + 6}{z^2 + 2z - 3}$ **16.** $\frac{5t^2 + 6t - 8}{3t^2 + 5t - 2}$ **17.** $\frac{32a^3}{16a^2 - 8a}$

18. $\frac{3z^2 + 12z}{z^4}$ **19.** $\frac{16 + 16m + 3m^2}{m^2 - 3m - 28}$ **20.** $\frac{10c + c^2 - 3c^3}{5c^2 - 6c - 8}$

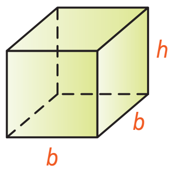
21. In the figure at the right, what is the ratio of the area of the shaded triangle to the area of the rectangle? Write your answer in simplified form.



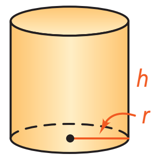
22. **Justify Mathematical Arguments (1)(G)** Is $\frac{x^2 - 9}{x + 3}$ the same as $x - 3$? Explain.

STEM **23. a.** **Apply Mathematics (1)(A)** To keep heating costs down for a building, architects want the ratio of surface area to volume to be as small as possible. What is an expression for the ratio of surface area to volume for each figure?

i. square prism



ii. cylinder



b. For each figure, what is the ratio of surface area to volume when $b = 12$ ft, $h = 18$ ft, and $r = 6$ ft?

24. **Explain Mathematical Ideas (1)(G)** A student simplified a rational expression as shown at the right. Describe and correct the error.

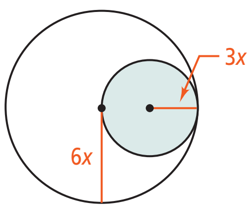
$$\frac{x^2 + 2x}{2x} = \frac{x^2 + \cancel{2x}}{\cancel{2x}} = x^2$$

25. **Apply Mathematics (1)(A)** A bank account with principal P earns interest at rate r (expressed as a decimal), compounded annually. What is the ratio of the balance after 3 yr to the balance after 1 yr?

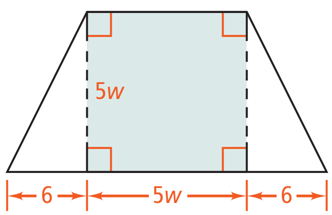
26. **Display Mathematical Ideas (1)(G)** Write a rational expression that has 4 and -3 as excluded values.

Write a ratio in simplified form of the area of the shaded figure to the area of the figure that encloses it.

27.



28.



Simplify each expression. State any excluded values.

29. $\dfrac{m^2 - n^2}{m^2 + 11mn + 10n^2}$

30. $\dfrac{a^2 - 5ab + 6b^2}{a^2 + 2ab - 8b^2}$

31. $\dfrac{36v^2 - 49w^2}{18v^2 + 9vw - 14w^2}$

Justify Mathematical Arguments (1)(G) **Determine whether each statement is *always, sometimes,* or *never* true for real numbers *a* and *b*. Explain.**

32. $\dfrac{2b}{b} = 2$

33. $\dfrac{ab^3}{b^4} = ab$

34. $\dfrac{a^2 + 6a + 5}{2a + 2} = \dfrac{a + 5}{2}$

TEXAS End-of-Course PRACTICE

35. Which expression simplifies to -1?

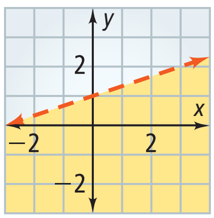
A. $\dfrac{x + 1}{x - 1}, x \neq 1$

B. $\dfrac{r + 3}{3 - r}, r \neq 3$

C. $\dfrac{n - 2}{2 - n}, n \neq 2$

D. $\dfrac{4 - p}{4 + p}, p \neq -4$

36. Which inequality represents the graph at the right?



F. $y > \frac{1}{3}x + 1$

G. $y < \frac{1}{3}x + 1$

H. $y \geq \frac{1}{3}x + 1$

J. $y \leq \frac{1}{3}x + 1$

37. What is $\dfrac{\sqrt{6}}{\sqrt{96}}$ in simplified form?

A. 16

B. 4

C. $\frac{1}{4}$

D. $\frac{1}{16}$

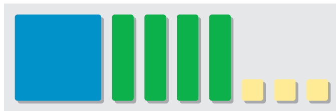
USE WITH LESSON 7-10 TEKS (10)(C), (1)(C)

You can use algebra tiles to model polynomial division.

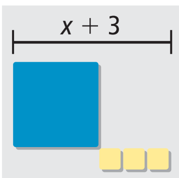
Activity

What is $(x^2 + 4x + 3) \div (x + 3)$? Use algebra tiles.

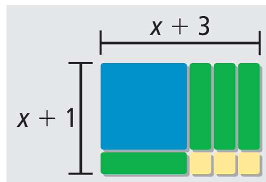
Step 1 Use algebra tiles to model the dividend, $x^2 + 4x + 3$.



Step 2 Use the x^2-tile and the 1-tiles to form a figure with length $x + 3$, the divisor.



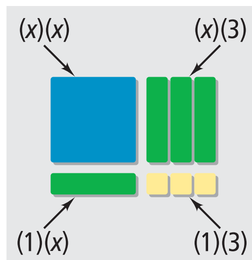
Step 3 Use the remaining tiles to fill in the rectangle.



Since $(x + 1)(x + 3) = x^2 + 4x + 3$, you can write $(x^2 + 4x + 3) \div (x + 3) = x + 1$.

Check Check your result by multiplying $x + 1$ and $x + 3$. The product should be the dividend, $x^2 + 4x + 3$.

$$\begin{aligned}(x+1)(x+3) &= (x)(x) + (x)(3) + (1)(x) + (1)(3)\\ &= x^2 + 3x + x + 3\\ &= x^2 + 4x + 3 \ ✔\end{aligned}$$



Exercises

Select Tools to Solve Problems (1)(C) Use algebra tiles to find each quotient. Check your result.

1. $(x^2 + 6x + 8) \div (x + 4)$

2. $(x^2 + 5x + 6) \div (x + 2)$

3. $(x^2 + 8x + 12) \div (x + 6)$

4. $(x^2 + 8x + 7) \div (x + 1)$

5. **Analyze Mathematical Relationships (1)(F)** In Exercises 1–4, the divisor is a factor of the dividend. How do you know? Can you use algebra tiles to represent polynomial division when the divisor is *not* a factor of the dividend? Explain.

7-10 Dividing Polynomials

TEKS FOCUS

TEKS (10)(C) Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(E)

VOCABULARY

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

ESSENTIAL UNDERSTANDING

You can divide polynomials using techniques similar to the techniques used for dividing real numbers, including long division.

take note

Concept Summary Dividing a Polynomial by a Polynomial

Step 1 Arrange the terms of the dividend and divisor in standard form. If a term is missing from the dividend, add the term with a coefficient of 0.

Step 2 Divide the first term of the dividend by the first term of the divisor. This is the first term of the quotient.

Step 3 Multiply the first term of the quotient by the whole divisor and place the product under the dividend.

Step 4 Subtract this product from the dividend.

Step 5 Bring down the next term.

Repeat Steps 2–5 as necessary until the degree of the remainder is less than the degree of the divisor.

Problem 1

Dividing by a Monomial

Plan

How can you change this problem into one you know how to solve?
Change the division to multiplication so that you can use the Distributive Property.

A **What is $(15x - 6) \div 3x$?**

$(15x - 6) \div 3x = (15x - 6) \cdot \frac{1}{3x}$ — Multiply by $\frac{1}{3x}$, the reciprocal of $3x$.

$= \frac{15x}{3x} - \frac{6}{3x}$ — Use the Distributive Property.

$= 5 - 2x^{-1}$ — Subtract exponents when dividing powers with the same base.

$= 5 - \frac{2}{x}$ — Simplify.

The quotient is $5 - \frac{2}{x}$.

B **What is $(9x^3 - 6x^2 + 15x) \div 3x^2$?**

$(9x^3 - 6x^2 + 15x) \div 3x^2 = (9x^3 - 6x^2 + 15x) \cdot \frac{1}{3x^2}$ — Multiply by $\frac{1}{3x^2}$, the reciprocal of $3x^2$.

$= \frac{9x^3}{3x^2} - \frac{6x^2}{3x^2} + \frac{15x}{3x^2}$ — Use the Distributive Property.

$= 3x^1 - 2x^0 + 5x^{-1}$ — Subtract exponents when dividing powers with the same base.

$= 3x - 2 + \frac{5}{x}$ — Simplify.

The quotient is $3x - 2 + \frac{5}{x}$.

Problem 2

TEKS Process Standard (1)(G)

Dividing by a Binomial

Plan

How do you get started?
Divide the first term in the dividend by the first term in the divisor. Here, you divide $3d^2$ by d.

A **What is $(3d^2 - 4d + 13) \div (d + 3)$?**

Step 1 Begin the long division process.

Align terms by their degrees. Put $3d$ above $-4d$ of the dividend.

$$\begin{array}{r} 3d \\ d+3\overline{)3d^2 - 4d + 13} \\ \underline{3d^2 + 9d} \\ -13d + 13 \end{array}$$

Divide: $3d^2 \div d = 3d$.
Multiply: $3d(d + 3) = 3d^2 + 9d$. Then subtract.
Bring down 13.

Step 2 Repeat the process: divide, multiply, subtract, and bring down.

Align terms by their degrees. Put -13 above 13 of the dividend.

$$\begin{array}{r} 3d - 13 \\ d+3\overline{)3d^2 - 4d + 13} \\ \underline{3d^2 + 9d} \\ -13d + 13 \\ \underline{-13d - 39} \\ 52 \end{array}$$

Divide: $-13d \div d = -13$.
Multiply: $-13(d+3) = -13d - 39$. Then subtract.
The remainder is 52.

The quotient is $3d - 13 + \frac{52}{d + 3}$.

continued on next page ▶

Problem 2 *continued*

B **What is $(20t^2 - 5t + 4) \div (4t^2 - t)$?**

$$\begin{array}{r@{}l}
 & 5 \\
4t^2 - t & \overline{)20t^2 - 5t + 4} \\
 & \underline{20t^2 - 5t\phantom{{}+4}} \\
 & \phantom{20t^2 - {}}0 + 4
\end{array}$$

Divide: $20t^2 \div 4t^2 = 5$

Multiply: $5(4t^2 - t)$

The remainder is 4.

The quotient is $5 + \frac{4}{4t^2 - t}$.

Problem 3

TEKS Process Standard (1)(E)

Dividing Polynomials With a Zero Coefficient

Geometry **The width w of a rectangle is $3z - 1$. The area A of the rectangle is $18z^3 - 8z + 2$. What is an expression for the length of the rectangle?**

Know

Area: $18z^3 - 8z + 2$

Width: $3z - 1$

Need

The length of the rectangle

Plan

Use the formula for the area of a rectangle, $A = \ell w$. Divide A by w to solve for ℓ.

Think

Why add a term with a coefficient of 0?

If the dividend is missing a term when written in standard form, you must add the term with a coefficient of 0 to act as a placeholder.

$$\begin{array}{r@{}l}
 & \phantom{)18z^3 + {}}6z^2 + 2z - 2 \\
3z - 1 & \overline{)18z^3 + 0z^2 - 8z + 2} \\
 & \underline{18z^3 - 6z^2} \\
 & \phantom{18z^3 - {}}6z^2 - 8z \\
 & \phantom{18z^3 - {}}\underline{6z^2 - 2z} \\
 & -6z + 2 \\
 & \underline{-6z + 2} \\
 & \phantom{18z^3 - 6z^2 - 6z + {}}0
\end{array}$$

The dividend has no z^2-term. So rewrite the dividend to include a z^2-term with coefficient 0.

An expression for the length of the rectangle is $6z^2 + 2z - 2$.

Problem 4

Reordering Terms and Dividing Polynomials

Think

How can you eliminate choices?
You can eliminate choice D since the denominator is incorrect.

Multiple Choice What is $(-10x - 1 + 4x^2) \div (-3 + 2x)$?

Ⓐ $2x - 2$

Ⓑ $2x - 2 - \frac{7}{2x - 3}$

Ⓒ $2x - 2 + \frac{7}{2x - 3}$

Ⓓ $2x - 2 + \frac{7}{2x + 2}$

$$\begin{array}{r}
2x - 2 \\
2x - 3\overline{)4x^2 - 10x - 1} \\
\underline{4x^2 - 6x} \\
-4x - 1 \\
\underline{-4x + 6} \\
-7
\end{array}$$

You must rewrite $-10x - 1 + 4x^2$ and $-3 + 2x$ in standard form before you divide.

The quotient is $2x - 2 - \frac{7}{2x - 3}$. The correct answer is B.

PRACTICE and APPLICATION EXERCISES

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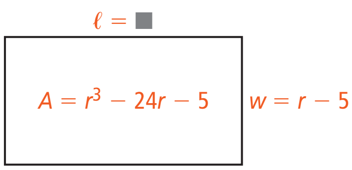
Divide.

1. $(x^6 - x^5 + x^4) \div x^2$
2. $(12x - 8) \div 4x$
3. $(8q^2 - 32q) \div 2q^2$
4. $(7t^5 + 14t^4 - 28t^3 + 35t^2) \div 7t^2$
5. $(n^2 - 5n + 4) \div (n - 4)$
6. $(y^2 - y + 2) \div (y + 2)$
7. $(3b^3 - 10b^2 + 4) \div (3b - 1)$
8. $(-16c^2 + 28c + 12) \div (4c^2 - 7c - 3)$

Write an expression for the missing dimension in each figure.

9.



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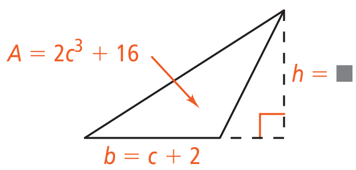


Divide.

11. $(49 + 16b + 2b^2) \div (2b + 4)$
12. $(4a^2 - 6 + 3a) \div (7 + 4a)$
13. $(-13x + 6x^3 - 6 - x^2) \div (3x - 5)$
14. $(6 - q + 8q^3 - 4q^2) \div (2q - 2)$
15. $(7b + 16b^3) \div (-1 + 8b)$
16. $(4y + 9y^3 - 7) \div (-5 + 3y)$

17. **Display Mathematical Ideas (1)(G)** Write a binomial and a trinomial using the same variable. Divide the trinomial by the binomial.

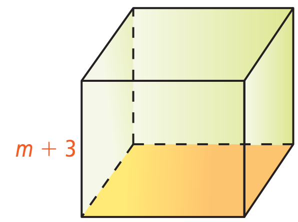
18. **Create Representations to Communicate Mathematical Ideas (1)(E)** The area A of a trapezoid is $x^3 + 2x^2 - 2x - 3$. The lengths of its two bases b_1 and b_2 are x and $x^2 - 3$, respectively. What is an expression for the height h of the trapezoid?

Divide.

19. $(3k^3 - 0.9k^2 - 1.2k) \div 3k$
20. $(-7s + 6s^2 + 5) \div (2s + 3)$
21. $(2t^4 - 2t^3 + 3t - 1) \div (2t^3 + 1)$
22. $(z^4 + z^2 - 2) \div (z + 3)$
23. $(-2z^3 - z + z^2 + 1) \div (z + 1)$
24. $(6m^3 + 3m + 70) \div (m + 4)$

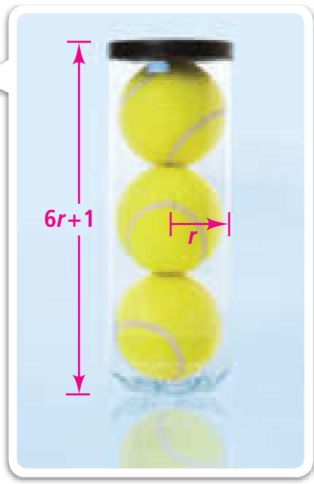
25. **Explain Mathematical Ideas (1)(G)** Suppose you divide a polynomial by a binomial. How do you know whether the binomial is a factor of the polynomial?

26. The volume of the rectangular prism shown at the right is $m^3 + 8m^2 + 19m + 12$. What is the area of the shaded base of the prism?



27. **Analyze Mathematical Relationships (1)(F)** Find a pattern by dividing the polynomials.
   a. What is $(d^2 - d + 1) \div (d + 1)$?
   b. What is $(d^3 - d^2 + d - 1) \div (d + 1)$?
   c. What is $(d^4 - d^3 + d^2 - d + 1) \div (d + 1)$?
   d. What do you think would be the result of dividing $d^5 - d^4 + d^3 - d^2 + d - 1$ by $d + 1$?
   e. Verify your prediction by dividing the polynomials.

28. **Apply Mathematics (1)(A)** Three tennis balls with radius r are packed into a cylindrical can with radius r and height $6r + 1$. What fraction of the can is empty? Write your answer in the form quotient + $\frac{\text{remainder}}{\text{divisor}}$.



29. Simplify $\frac{x^{16} - 1}{x - 1}$ by long division and by factoring. Which method do you prefer? Explain your answer.

30. **Apply Mathematics (1)(A)** One way to measure a business's efficiency is by dividing the business's revenue by its expenses. The annual revenue, in millions of dollars, of a certain airline can be modeled by $200s^3 - s^2 + 400s + 1500$, where s is the number of passengers, in hundreds of thousands. The expenses, in millions of dollars, of the airline can be modeled by $200s + 300$. What is the airline's revenue divided by its expenses? Write your answer in the form quotient + $\frac{\text{remainder}}{\text{divisor}}$.

31. **Analyze Mathematical Relationships (1)(F)** If $x + 3$ is a factor of $x^2 - x - k$, what is the value of k?

32. **Apply Mathematics (1)(A)** Consider the formula for distance traveled, $d = rt$.

  a. Solve the formula for t.

  b. Use your answer from part (a). What is an expression for the time it takes to travel a distance of $t^3 - 6t^2 + 5t + 12$ miles at a rate of $t + 1$ miles per hour?

Divide.

33. $(4a^3b^4 - 6a^2b^5 + 10a^2b^4) \div 2ab^2$

34. $(15x^2 + 7xy - 2y^2) \div (5x - y)$

35. $(90r^6 + 28r^5 + 45r^3 + 2r^4 + 5r^2) \div (9r + 1)$

36. $(2b^6 + 2b^5 - 4b^4 + b^3 + 8b^2 - 3) \div (b^3 + 2b^2 - 1)$

TEXAS End-of-Course PRACTICE

37. Which of the following is true for $(2x^2 + 4x + 2) \div 2x$?

  I. The remainder is negative.

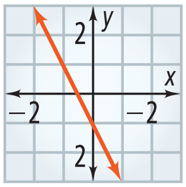
  II. The dividend is in standard form.

  III. The quotient is greater than the divisor for positive values of x.

  A. I only **C.** I and II

  B. II only **D.** II and III

38. Which equation represents the line that passes through $(5, -8)$ and is parallel to the line at the right?



  F. $y = 2x + 2$ **H.** $y = -2x$

  G. $y + 2x = 2$ **J.** $y - 2x = 2$

39. What are the factors of the expression $x^3 - 4x$?

  A. $x^3, -4x$ **C.** $x - 2, x + 2$

  B. $x, x^2 - 4$ **D.** $x, x - 2, x + 2$

40. A theater has 18 rows of seats. Each row has 28 seats. Tickets cost \$4 for adults and \$2.50 for children. The Friday night show was sold out and the revenue from ticket sales was \$1935. Barbara says that 445 adults were at the show. Is her statement reasonable? Explain your answer.

Topic 7 Review

TOPIC VOCABULARY

- binomial, *p. 268*
- degree of a monomial, *p. 268*
- degree of a polynomial, *p. 268*
- difference of two squares, *p. 296*
- excluded value, *p. 305*
- factoring by grouping, *p. 301*
- monomial, *p. 268*
- perfect-square trinomial, *p. 296*
- polynomial, *p. 268*
- rational expression, *p. 305*
- standard form of a polynomial, *p. 268*
- trinomial, *p. 268*

Check Your Understanding

Choose the correct term to complete each sentence.

1. A polynomial that has two terms is a(n) __?__ .

2. A monomial or the sum of two or more monomials is a(n) __?__ .

3. A(n) __?__ is an expression that is a number, a variable, or a product of a number and one or more variables.

4. A polynomial that is the product of two identical binomial factors is a(n) __?__ .

5. The sum of the exponents of the variables in a monomial is the __?__ .

7-1 Adding and Subtracting Polynomials

Quick Review

A **monomial** is a number, a variable, or a product of a number and one or more variables. A **polynomial** is a monomial or the sum of two or more monomials. The **degree of a polynomial** in one variable is the same as the degree of the monomial with the greatest exponent. To add two polynomials, add the like terms of the polynomials. To subtract a polynomial, add the opposite of the polynomial.

Example

What is the difference of $3x^3 - 7x^2 + 5$ and $2x^2 - 9x - 1$?

$$\begin{aligned}
&(3x^3 - 7x^2 + 5) - (2x^2 - 9x - 1)\\
&\quad = 3x^3 - 7x^2 + 5 - 2x^2 + 9x + 1\\
&\quad = 3x^3 + (-7x^2 - 2x^2) + 9x + (1 + 5)\\
&\quad = 3x^3 - 9x^2 + 9x + 6
\end{aligned}$$

Exercises

Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms.

6. $4r + 3 - 9r^2 + 7r$

7. $3 + b^3 + b^2$

8. $3 + 8t^2$

9. $n^3 + 4n^5 + n - n^3$

10. $7x^2 + 8 + 6x - 7x^2$

11. p^3q^3

Simplify. Write each answer in standard form.

12. $(2v^3 - v + 8) + (-v^3 + v - 3)$

13. $(6s^4 + 7s^2 + 7) + (8s^4 - 11s^2 + 9s)$

14. $(4h^3 + 3h + 1) - (-5h^3 + 6h - 2)$

15. $(8z^3 - 3z^2 - 7) - (z^3 - z^2 + 9)$

7-2 Multiplying and Factoring

Quick Review

You can multiply a monomial and a polynomial using the Distributive Property. You can factor a polynomial by finding the greatest common factor (GCF) of the terms of the polynomial.

Example

What is the factored form of $10y^4 - 12y^3 + 4y^2$?

First find the GCF of the terms of the polynomial.

$$10y^4 = 2 \cdot 5 \cdot y \cdot y \cdot y \cdot y$$
$$12y^3 = 2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot y$$
$$4y^2 = 2 \cdot 2 \cdot y \cdot y$$

The GCF is $2 \cdot y \cdot y$, or $2y^2$.

Then factor out the GCF.

$$10y^4 - 12y^3 + 4y^2 = 2y^2(5y^2) + 2y^2(-6y) + 2y^2(2)$$
$$= 2y^2(5y^2 - 6y + 2)$$

Exercises

Simplify each product. Write in standard form.

16. $5k(3 - 4k)$
17. $4m(2m + 9m^2 - 6)$
18. $6g^2(g - 8)$
19. $3d(6d + d^2)$
20. $-2n^2(5n - 9 + 4n^2)$
21. $q(11 + 8q - 2q^2)$

Find the GCF of the terms of each polynomial. Then factor the polynomial.

22. $12p^4 + 16p^3 + 8p$
23. $3b^4 - 9b^2 + 6b$
24. $45c^5 - 63c^3 + 27c$
25. $4g^2 + 8g$
26. $3t^4 - 6t^3 - 9t + 12$
27. $30h^5 - 6h^4 - 15h^3$

28. The GCF of two numbers p and q is 5. Can you find the GCF of $6p$ and $6q$? Explain your answer.

7-3 and 7-4 Multiplying Binomials and Multiplying Special Cases

Quick Review

You can use algebra tiles, tables, or the Distributive Property to multiply polynomials. The FOIL method (First, Outer, Inner, Last) can be used to multiply two binomials. You can also use rules to multiply special case binomials.

Example

What is the simplified form of $(4x + 3)(3x + 2)$?

Use FOIL to multiply the binomials. Find the product of the first terms, the outer terms, the inner terms, and the last terms. Then add.

$$(4x + 3)(3x + 2) = (4x)(3x) + (4x)(2) + (3)(3x) + (3)(2)$$
$$= 12x^2 + 8x + 9x + 6$$
$$= 12x^2 + 17x + 6$$

Exercises

Simplify each product. Write in standard form.

29. $(w + 1)(w + 12)$
30. $(2s - 3)(5s + 4)$
31. $(3r - 2)^2$
32. $(6g + 7)(g - 8)$
33. $(7q + 2)(3q + 8)$
34. $(4n^3 + 5)(3n + 5)$
35. $(t + 9)(t - 3)$
36. $(6c + 5)^2$
37. $(7h - 3)(7h + 3)$
38. $(y - 6)(3y + 7)$
39. $(4a - 7)(8a + 3)$
40. $(4b - 3)(4b + 3)$

41. A rectangle has dimensions $3x + 5$ and $x + 7$. Write an expression for the area of the rectangle as a product and as a polynomial in standard form.

7-5 and 7-6 Factoring Quadratic Trinomials

Quick Review

You can write some quadratic trinomials as the product of two binomial factors. When you factor a polynomial, be sure to factor out the GCF first.

Example

What is the factored form of $x^2 + 7x + 12$?

List the pairs of factors of 12. Identify the pair with a sum of 7.

Factors of 12	Sum of Factors
1, 12	13
2, 6	8
3, 4	7 ✔

$x^2 + 7x + 12 = (x + 3)(x + 4)$

Exercises

Factor each expression.

42. $g^2 - 5g - 14$

43. $2n^2 + 3n - 2$

44. $6k^2 - 10k\ell + 4\ell^2$

45. $p^2 + 8p + 12$

46. $r^2 + 6r - 40$

47. $6m^2 + 25mn + 11n^2$

48. $t^2 - 13t - 30$

49. $2g^2 - 35g + 17$

50. $3x^2 + 3x - 6$

51. $d^2 - 18d + 45$

52. $w^2 - 15w - 54$

53. $21z^2 - 70z + 49$

54. $-2h^2 + 4h + 70$

55. $x^2 + 21x + 38$

56. $10v^2 + 11v - 8$

57. $5g^2 + 15g + 10$

58. Can you factor the expression $2x^2 + 15x + 9$? Explain why or why not.

7-7 Factoring Special Cases

Quick Review

When you factor a perfect-square trinomial, the two binomial factors are the same.

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

When you factor a difference of squares of two terms, the two binomial factors are the sum and the difference of the two terms.

$$a^2 - b^2 = (a + b)(a - b)$$

Example

What is the factored form of $81t^2 - 90t + 25$?

First, rewrite the first and last terms as squares. Then, determine whether the middle term equals $-2ab$.

$$\begin{aligned} 81t^2 - 90t + 25 &= (9t)^2 - 90t + 5^2 \\ &= (9t)^2 - 2(9t)(5) + 5^2 \\ &= (9t - 5)^2 \end{aligned}$$

Exercises

Factor each expression.

59. $s^2 - 20s + 100$

60. $16q^2 + 56q + 49$

61. $r^2 - 64$

62. $9z^2 - 16$

63. $25m^2 + 80m + 64$

64. $49n^2 - 4$

65. $g^2 - 225$

66. $9p^2 - 42p + 49$

67. $36h^2 - 12h + 1$

68. $w^2 + 24w + 144$

69. $32v^2 - 8$

70. $25x^2 - 36$

71. Find an expression for the length of a side of a square with an area of $9n^2 + 54n + 81$.

72. Suppose you are using algebra tiles to factor a quadratic trinomial. What do you know about the factors of the trinomial when the tiles form a square?

7-8 Factoring by Grouping

Quick Review

When a polynomial has four or more terms, you may be able to group the terms and find a common binomial factor. Then you can use the Distributive Property to factor the polynomial.

Example

What is the factored form of $2r^3 - 12r^2 + 5r - 30$?

First, factor out the GCF from each group of two terms. Then, factor out a common binomial factor.

$$\begin{aligned} 2r^3 - 12r^2 + 5r - 30 &= 2r^2(r-6) + 5(r-6) \\ &= \left(2r^2+5\right)(r-6) \end{aligned}$$

Exercises

Find the GCF of the first two terms and the GCF of the last two terms for each polynomial.

73. $6y^3 - 3y^2 + 2y - 1$

74. $8m^3 + 40m^2 + 3m + 15$

Factor completely.

75. $6d^4 + 4d^3 - 6d^2 - 4d$

76. $11b^3 - 6b^2 + 11b - 6$

77. $45z^3 + 20z^2 + 9z + 4$

78. $9a^3 - 12a^2 + 18a - 24$

7-9 Simplifying Rational Expressions

Quick Review

A **rational expression** is an expression that can be written in the form $\frac{\text{polynomial}}{\text{polynomial}}$. A rational expression is in simplified form when the numerator and denominator have no common factors other than 1.

Example

What is the simplified form of $\frac{x^2-9}{x^2-2x-15}$?

$$\frac{x^2-9}{x^2-2x-15} = \frac{(x+3)(x-3)}{(x+3)(x-5)} \quad \text{Factor the numerator and denominator.}$$

$$= \frac{\cancel{(x+3)}^{1}(x-3)}{{}_{1}\cancel{(x+3)}(x-5)} \quad \text{Divide out the common factor.}$$

$$= \frac{x-3}{x-5} \quad \text{Simplify.}$$

Exercises

Simplify each expression. State any excluded values.

79. $\frac{2x^2+6x}{10x^3}$

80. $\frac{m-3}{3m-9}$

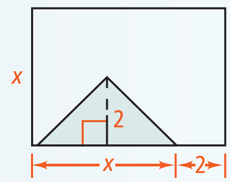
81. $\frac{x^2+6x+9}{5x+15}$

82. $\frac{2a^2-4a+2}{3a^2-3}$

83. $\frac{2s^2-5s-12}{2s^2-9s+4}$

84. $\frac{4-c}{2c-8}$

85. What fraction of the rectangle is shaded? Write your answer as a rational expression in simplified form.



7-10 Dividing Polynomials

Quick Review

You can divide rational expressions using the same properties you use to divide numerical fractions.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \text{ where } b \neq 0,\ c \neq 0, \text{ and } d \neq 0.$$

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. To divide a polynomial by another polynomial, use long division. When dividing polynomials, write the answer as quotient + $\frac{\text{remainder}}{\text{divisor}}$.

Example

What is the quotient $(3x^2 - 10x + 3) \div (x - 3)$?

$$\begin{array}{r}
3x - 1 \\
x - 3\,\overline{)\,3x^2 - 10x + 3} \\
\underline{3x^2 - 9x} \\
-x + 3 \\
\underline{-x + 3} \\
0
\end{array}$$

The quotient is $3x - 1$.

Exercises

Divide.

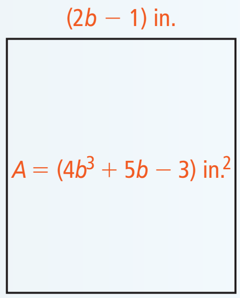
86. $(25n^3 - 11n + 4) \div (5n + 4)$

87. $(-16a - 15 + 15a^2) \div (3 + 5a)$

88. $(12x^2 + 9x - 7) \div 3x$

89. $(3d^2 + 2d - 29) \div (d + 3)$

90. The width and area of a rectangle are shown in the figure at the right. What is the length of the rectangle?



Topic 7 TEKS Cumulative Practice

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. The area of a rectangle is $6n^2 + n - 2$. Which expression could represent the perimeter of the rectangle?

A. $2n - 1$ **C.** $5n + 1$

B. $3n + 2$ **D.** $10n + 2$

2. A company sells calculators for \$35 each. Businesses must order a minimum of 100 calculators, and they must pay a shipping cost of \$50. Which amount of money represents a reasonable sum that a business might spend to purchase calculators?

F. \$750 **H.** \$2990

G. \$1070 **J.** \$3585

3. The formula for the volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base of the pyramid and h is the height of the pyramid. Which equation represents the height of the pyramid in terms of V and B?

A. $h = \frac{3V}{B}$ **C.** $h = 3VB$

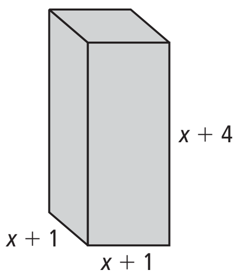
B. $h = \frac{V}{3B}$ **D.** $h = \frac{B}{3V}$

4. You are using a map to find the distance between your house and a friend's house. On the map, the distance is 2.5 in. Suppose the map's scale is $\frac{1}{8}$ in. = 1.5 mi. How far do you live from your friend?

F. 0.08 mi **H.** 3.75 mi

G. 0.2 mi **J.** 30 mi

5. Which expression represents the volume of the prism?



A. $x^3 + 4$

B. $x^3 + 6x^2 + 9x + 4$

C. $x^3 + 9x^2 + 6x + 4$

D. $x^3 + 4x^2 + x + 4$

6. Which equation models a line with positive slope and a positive x-intercept?

F. $5x - 2y = 14$

G. $-5x - 2y = 14$

H. $-5x + 2y = 14$

J. $5x + 2y = -14$

7. You can represent the width of a certain rectangle with the expression $x + 2$. The length of the rectangle is twice the width. What is the area of the rectangle?

A. $2x + 4$ **C.** $2x^2 + 8x + 8$

B. $2x^2 + 8$ **D.** $4x^2 + 16x + 16$

8. Which equation represents a line with slope that is greater than the slope of the line with equation $y = \frac{3}{4}x - 1$?

F. $y = -\frac{3}{4}x - 2$ **H.** $y = \frac{2}{3}x - 1$

G. $y = \frac{4}{3}x - 2$ **J.** $y = \frac{3}{4}x + 2$

9. What is the x-intercept of the line that passes through $(0, -4)$ and $(1, 4)$?

A. -4 **C.** $\frac{1}{2}$

B. $\frac{1}{4}$ **D.** 2

10. Megan earns \$20,000 per year plus 5% commission on her sales. Laurie earns \$32,000 per year plus 1% commission on her sales. Which system of equations can you use to determine the amounts that Megan and Laurie must sell s to receive equal pay p?

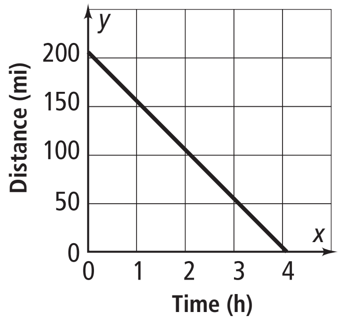
F. $p = 5s + 20{,}000$
$p = s + 32{,}000$

G. $p + 5s = 20{,}000$
$p + s = 32{,}000$

H. $p + 0.5s = 20{,}000$
$p + 0.1s = 32{,}000$

J. $p = 0.05s + 20{,}000$
$p = 0.01s + 32{,}000$

11. Your family is driving to the beach. The graph at the right relates your distance from the beach to the amount of time you spend driving.



What does the y-intercept of the graph represent?

A. a stop on the way to the beach

B. your average speed in miles per hour

C. your distance from the beach before you started driving

D. the amount of time it takes to get to the beach

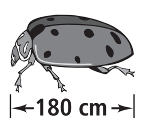
Gridded Response

12. Suppose $b = 2a - 16$ and $b = a + 2$. What is the value of a?

13. A student's score on a history test varies directly with the number of questions the student correctly answers. A student who correctly answers 14 questions receives a score of 70. What score would a student receive for correctly answering 15 questions?

14. You bought a candlestick holder for \$11.78 and several candles for \$.62 each. You spent a total of \$18.60. How many candles did you buy?

15. An artist is making a scale model of a ladybug for an insect museum. What is the length in millimeters of the actual ladybug that the artist is using to make this model?



Scale: 15 cm = 1 mm

16. A flagpole casts a shadow that is 9.1 m long. At the same time, a meter stick casts a shadow that is 1.4 m long. How tall is the flagpole in meters?

17. Line m passes through $(-9, 4)$ and $(9, 6)$. What is the y-intercept of line m?

18. Suppose you have \$200. Sweaters cost \$45 each. What is the greatest number of sweaters you can buy?

Constructed Response

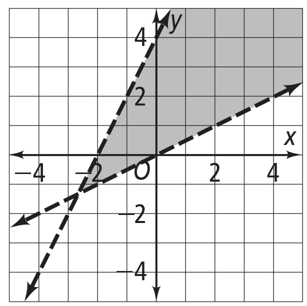
19. What is the seventh term in the following sequence?

$$81, 27, 9, 3, 1, \ \ldots$$

20. The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of its two bases.

Suppose the height of a trapezoid is $x - 2$. The lengths of the bases are $x + 2$ and $3x - 2$. What polynomial in standard form represents the area of the trapezoid? Show your work.

21. Write a system of inequalities for the graph.



22. Write the expression $\sqrt[3]{8a^2}$ using rational exponents.

23. Simplify the expression $(3x - 5)(2x + 5)$.

24. You sell hot dogs and sodas at basketball games. Each hot dog costs \$1.50 and each soda costs \$.50. At one game, you made a total of \$78.50. In all, you sold 87 hot dogs and sodas.

a. Write a system of linear equations to solve the problem.

b. How many hot dogs did you sell? How many sodas did you sell?

25. If $A(2, 1)$, $B(5, 4)$, $C(8, 12)$, and $D(5, 9)$ are the coordinates of the vertices of a quadrilateral, verify that the quadrilateral $ABCD$ is a parallelogram.

Topic 5

Lessons 5-1 to 5-4

Simplify each expression. Use only positive exponents.

1. $(2t)^{-6}$
2. $5m^5m^{-8}$
3. $(4.5)^4(4.5)^{-2}$
4. $(m^7t^{-5})^2$
5. $(x^2n^4)(n^{-8})$
6. $(w^{-2}j^{-4})^{-3}(j^7j^3)$
7. $(t^6)^3(m)^2$
8. $(3n^4)^2$
9. $\frac{r^5}{g^{-3}}$
10. $\frac{1}{a^{-4}}$
11. $\frac{w^7}{w^{-6}}$
12. $\frac{6}{t^{-4}}$
13. $\frac{a^2b^{-7}c^4}{a^5b^3c^{-2}}$
14. $\frac{(2t^5)^3}{4t^8t^{-1}}$
15. $\left(\frac{a^6}{a^7}\right)^{-3}$
16. $\left(\frac{c^5c^{-3}}{c^{-4}}\right)^{-2}$
17. $\left(\frac{4x^3}{8x^{-2}}\right)^0$
18. $\left(\frac{y^{-3}}{y^3}\right)^2$

Evaluate each expression for $m = 2$, $t = -3$, $w = 4$, and $z = 0$.

19. t^m
20. t^{-m}
21. $(w \cdot t)^m$
22. $w^m \cdot t^m$
23. $(w^z)^m$
24. w^mw^z
25. $z^{-t}(m^t)^z$
26. $w^{-t}t^t$
27. $\left(\frac{t^w}{m^t}\right)^z$

28. Suppose an investment doubles in value every 5 years. This year the investment is worth \$12,480. How much will it be worth 10 years from now? How much was it worth 5 years ago?

29. What is the volume of a cube with a side length of $\frac{4}{5}$ m?

30. A light-year is the distance light travels in one year. If the speed of light is about 3×10^5 km/s, how long is a light-year in kilometers? (Use 365 days for the length of a year.)

31. The radius of Earth is approximately 6.4×10^6 m. Use the formula $V = \frac{4}{3}\pi r^3$ to find the volume of Earth.

32. A spherical cell has a radius of 2.75×10^{-6} m. Use the formula for the surface area of a sphere, $S.A. = 4\pi r^2$, to find the surface area of a cell.

33. The speed of sound is approximately 1.2×10^3 km/h. How long does it take for sound to travel 7.2×10^2 km? Write your answer in minutes.

Lesson 5-5

Find the value of each expression.

34. $\sqrt[2]{64}$
35. $\sqrt[3]{343}$
36. $\sqrt[4]{16}$
37. $\sqrt[3]{125}$
38. $\sqrt[4]{256}$
39. $\sqrt[2]{144}$

Write each expression in radical form.

40. $b^{\frac{3}{4}}$
41. $16a^{\frac{2}{3}}$
42. $(4c)^{\frac{1}{2}}$
43. $y^{\frac{1}{4}}$
44. $(32b)^{\frac{2}{3}}$
45. $12a^{\frac{3}{4}}$

Lesson 5-5 *continued*

Write each expression in exponential form.

46. $\sqrt[4]{n^3}$
47. $\sqrt[3]{27m^2}$
48. $\sqrt{81z}$
49. $\sqrt[3]{128y^2}$
50. $\sqrt{(5b)^4}$
51. $\sqrt[4]{(16x)^2}$

Lesson 5-6

Simplify each radical expression.

52. $\frac{\sqrt{27}}{\sqrt{81}}$
53. $\sqrt{\frac{25}{4}}$
54. $\sqrt{\frac{50}{9}}$
55. $\frac{\sqrt{72}}{\sqrt{50}}$
56. $\sqrt{25} \cdot \sqrt{4}$
57. $\sqrt{27} \cdot \sqrt{3}$
58. $\sqrt{\frac{44x^4}{11}}$
59. $\frac{\sqrt{3c^2}}{\sqrt{27}}$
60. $\sqrt{45} \cdot \sqrt{18}$

Use the formula $d = \sqrt{1.5h}$ to estimate the distance d in miles to a horizon when h is the height of the viewer's eyes above the ground in feet. Round your answer to the nearest mile.

61. Find the distance you can see to the horizon from the top of a building that is 355 ft high.
62. Find the distance you can see to the horizon from an airplane that is 36,000 ft above the ground.
63. Find the distance you can see to the horizon from 15 ft above the ground.

Lesson 5-7

Determine whether the given lengths can be side lengths of a right triangle.

64. 15, 36, 39
65. 3, 7, 10
66. 8, 15, 17
67. $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$
68. 6, 7, 8
69. 12, 16, 20

For the values given, a and b are legs of a right triangle. Find the length of the hypotenuse. If necessary, round to the nearest tenth.

70. $a = 6$, $b = 8$
71. $a = 5$, $b = 9$
72. $a = 4$, $b = 10$
73. $a = 9$, $b = 1$
74. $a = 7$, $b = 3.5$
75. $a = 1.4$, $b = 2.3$

Use the Pythagorean theorem to answer each question.

76. A 20-ft ladder is placed 5 ft from the base of a building. How high on the building will the ladder reach?
77. A soccer field is 80 yd long and 35 yd wide. What is the diagonal distance across the field?

Topic 6

Lesson 6-1

Describe a pattern in each sequence. What are the next two terms of each sequence?

1. 1, 5, 9, 13, . . .

2. 1.2, 3.6, 10.8, 32.4, . . .

3. 16, 4, 1, $\frac{1}{4}$, . . .

Tell whether the sequence is arithmetic. If it is, identify the common difference.

4. −3, −5, −7, −9, . . .

5. 5, 12, 19, 26, . . .

6. $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{2.5}$, $\frac{1}{1.25}$, . . .

7. 6.5, 6.25, 6.0, 5.75, . . .

8. 1, 2, 4, 8, . . .

9. $\frac{3}{5}$, $\frac{8}{5}$, $\frac{13}{5}$, $\frac{18}{5}$, . . .

Tell whether the sequence is geometric. If it is, identify the common ratio.

10. 2, 10, 50, 250, . . .

11. 7, 15, 23, 31, . . .

12. 3, 12, 48, 192, . . .

13. −1, 7, −49, 343, . . .

14. 48, 24, 12, 6, . . .

15. 17, 15, 13, 11, . . .

Write an explicit formula for the *n*th term of each arithmetic sequence. Find the 6th and 9th terms of each sequence.

16. −1, 3, 7, 11, . . .

17. 9, 15, 21, 27, . . .

18. 5, $\frac{9}{2}$, 4, $\frac{7}{2}$, . . .

Write an explicit formula for the *n*th term of each geometric sequence. Find the 5th and 9th terms of each sequence.

19. 8, 24, 72, 216, . . .

20. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{8}{3}$, . . .

21. 0.4, 0.2, 0.1, 0.05, . . .

Lesson 6-2

Write a recursive definition for each arithmetic sequence.

22. 54, 59, 64, 69, . . .

23. 0.1, 0.5, 0.9, 1.3, . . .

24. $\frac{10}{3}$, 4, $\frac{14}{3}$, $\frac{16}{3}$, . . .

25. 10, 4, −2, −8, . . .

26. $\frac{1}{2}$, 0, $-\frac{1}{2}$, −1, . . .

27. 0.25, −0.75, −1.75, −2.75, . . .

Use the given recursive definition to find the first five terms of the arithmetic sequence it defines.

28. $A(n) = A(n-1) + 8$; $A(1) = -5$

29. $A(n) = A(n-1) - \frac{2}{5}$; $A(1) = 6$

30. $A(n) = A(n-1) + 0.55$; $A(1) = 0.55$

31. $A(n) = A(n-1) - \frac{1}{4}$; $A(1) = \frac{1}{8}$

Lesson 6-2 *continued*

Write an explicit formula from each given recursive definition.

32. $A(n) = A(n-1) + 2.5$; $A(1) = 3$

33. $A(n) = A(n-1) + \frac{1}{8}$; $A(1) = \frac{1}{2}$

34. $A(n) = A(n-1) - 5$; $A(1) = -0.2$

35. $A(n) = A(n-1) + 0.4$; $A(1) = -0.1$

Write a recursive definition from each given explicit formula.

36. $A(n) = -5 + (n-1)(6)$

37. $A(n) = 2.3 + (n-1)(4.5)$

38. $A(n) = \frac{6}{7} + (n-1)(3)$

39. $A(n) = -2 + (n-1)\left(\frac{1}{2}\right)$

Lesson 6-3

Write a recursive definition for each geometric sequence.

40. 4, 20, 100, 500, . . .

41. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

42. 0.3, 0.18, 0.108, 0.0648, . . .

43. $\frac{5}{9}, \frac{5}{3}, 5, 15, \ldots$

44. $-\frac{1}{7}, -2, -28, -392, \ldots$

45. $-3, -\frac{12}{5}, -\frac{48}{25}, -\frac{192}{125}, -\frac{768}{625}, \ldots$

Find the indicated term of the geometric sequence described by the recursive definition.

46. $A(n) = A(n-1) \cdot 3$, where $A(1) = 6$; $A(5)$

47. $A(n) = A(n-1) \cdot \frac{1}{6}$, where $A(1) = 2592$; $A(7)$

48. $A(n) = A(n-1) \cdot 8$, where $A(1) = 0.25$; $A(5)$

49. A cell in a fly embryo takes about 8 minutes to divide (resulting in two cells). If the embryo has 64 cells, how many cells will it have after 40 minutes?

50. A geometric sequence has an initial value of 2 and a common ratio of 4. Write a recursive definition to represent this sequence.

51. A geometric sequence has an initial value of 5 and a common ratio of 10. Write a recursive definition to represent this sequence.

52. Write the recursive definition that represents the sequence shown in the table below.

n	1	2	3	4
$A(n)$	5	25	125	625

Topic 7

Lesson 7-1

Write each polynomial in standard form. Then name each polynomial based on degree and number of terms.

1. $4x + x^2 - 1$ **2.** $2n^2 + 3n^3$ **3.** $-4y$ **4.** $14d - d^4 + 3d$

Simplify. Write each answer in standard form.

5. $(6v + 3v^2 - 9v^3) + (7v - 4v^2 - 10v^3)$

6. $(-7c^3 + c^2 - 8c - 11) - (3c^3 + 2c^2 + c - 4)$

7. The sides of a rectangle are $4t - 1$ and $5t + 9$. Write an expression for the perimeter of the rectangle.

8. Find an expression for the perimeter of a triangle with side lengths $2t^2$, $4t - 3$, and $10 - 2t$.

Lesson 7-2

Simplify each product.

9. $2y(y + 1)$ **10.** $4b(b^2 + 3)$ **11.** $-3x(x^2 + 3x - 1)$ **12.** $2m^2(m^3 + m - 2)$

Find the GCF terms of each polynomial. Factor.

13. $3y^4 - 9y^2$ **14.** $t^6 + t^4 - t^5 + t^2$ **15.** $8v^6 + 2v^5 - 10v^9$ **16.** $6n^2 - 3n^3 + 2n^4$

17. A cylinder has a base area of $3w^2 + 5$ and a height of $4w$. Find an expression for the volume.

Lessons 7-3 and 7-4

Simplify each product. Write in standard form.

18. $(5c + 3)(-c + 2)$ **19.** $(3t - 1)(2t + 1)$ **20.** $(w + 2)(w^2 + 2w - 1)$

21. $(3x + 1)^2$ **22.** $(5t + 4)^2$ **23.** $(w - 1)(w^2 + w + 1)$

24. A circular pool has a radius of $5p - 3$ m. Write an expression for the area of the pool.

25. An office building has a rectangular base with side lengths $12y - 7$ and $22y + 4$. Write an expression for the area of the floor in the office building.

Lessons 7-5 to 7-7

Factor each expression.

26. $x^2 - 4x + 3$ **27.** $3x^2 - 4x + 1$ **28.** $4m^2 - 121$

29. $4g^2 + 4g + 1$ **30.** $-w^2 + 5w - 4$ **31.** $9t^2 + 12t + 4$

Lessons 7-5 to 7-7 *continued*

Use factoring to find expressions for possible dimensions of each figure.

32. A circular window has an area of $49\pi v^2 + 84\pi v + 36\pi$.

33. A rectangular field has an area of $64m^2 - 169n^2$.

34. A rectangular prism has a volume of $6t^3 + 44t^2 + 70t$.

Lesson 7-8

Factor each expression.

35. $3y^3 + 9y^2 - y - 3$

36. $3u^3 + u^2 - 6u - 2$

37. $w^3 - 3w^2 + 3w - 9$

38. $4z^3 + 2z^2 - 2z - 1$

39. $3x^3 + 8x^2 - 3x$

40. $y^5 - 9y$

Use factoring to find expressions for possible dimensions of each figure.

41. A rectangular field has an area of $10k^3 + 25k^2 - 6k - 15$.

42. A rectangular swimming pool has an area of $5x^3 + 5x^2 - 2x - 2$.

43. A rectangular sheet of paper has an area of $6n^3 - 9n^2 - 8n + 12$.

Lesson 7-9

Simplify each expression. State any excluded values.

44. $\frac{a^2 + 2a - 3}{a + 3}$

45. $\frac{x + 7}{x^2 + 8x + 7}$

46. $\frac{t - 4}{4 - t}$

47. $\frac{m^2 + 7m + 10}{m^2 + 8m + 15}$

48. $\frac{6b^2 + 42b}{b^3}$

49. $\frac{x^2 - x - 6}{x^2 + 7x + 10}$

The expression $\frac{30rh}{r + h}$ gives the approximate baking time for bread in minutes based on the radius *r* and the height *h* of the baked loaf in inches. Use this expression to find the baking time for each loaf.

50. a roll that will have a radius of 2 in. and a height of 1.5 in.

51. a round loaf that will rise to 6 in. and have a radius of 4 in.

Lesson 7-10

Divide.

52. $(2x^3 - x^2 - 13x - 6) \div (x - 3)$

53. $(3x^3 - 3) \div (x - 1)$

54. $(3x^3 + 5x^2 - 22x + 24) \div (x + 4)$

55. The volume a prism is equal to the area of its base times its height. What is the area of the base of a prism with a volume of $15x^3 - 26x^2 + 43x - 14$ and a height of $5x - 2$?