PEARSON TEXAS ALGEBRA I

STUDENT TEXT AND HOMEWORK HELPER

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ISBN-13: 978-0-13-330063-5
ISBN-10: 0-13-330063-3
1 2 3 4 5 6 7 8 9 10 V0YJ 20 19 18 17 16 15 14

TOPIC 8 Quadratic Functions and Equations

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS (TEKS) FOCUS

(6)(A) Determine the domain and range of quadratic functions and represent the domain and range using inequalities.

(6)(B) Write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$).

(6)(C) Write quadratic functions when given real solutions and graphs of their related equations.

(7)(A) Graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

(7)(B) Describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions.

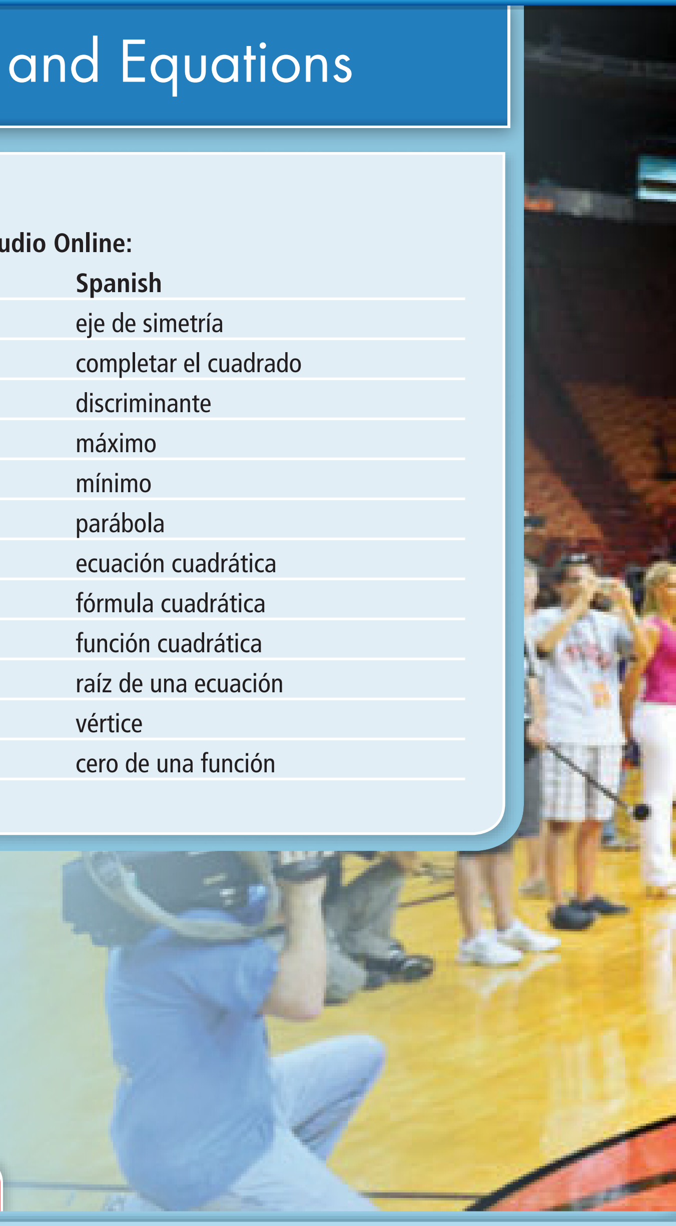
(7)(C) Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

(8)(A) Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

(8)(B) Write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(12)(B) Evaluate functions, expressed in function notation, given one or more elements in their domains.

Topic 8 Quadratic Functions and Equations



TOPIC OVERVIEW

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
axis of symmetry, *p. 327*	eje de simetría
completing the square, *p. 367*	completar el cuadrado
discriminant, *p. 372*	discriminante
maximum, *p. 327*	máximo
minimum, *p. 327*	mínimo
parabola, *p. 327*	parábola
quadratic equation, *p. 350*	ecuación cuadrática
quadratic formula, *p. 372*	fórmula cuadrática
quadratic function, *p. 326*	función cuadrática
root of an equation, *p. 350*	raíz de una ecuación
vertex, *p. 327*	vértice
zero of a function, *p. 350*	cero de una función

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3-Act Math

The Long Shot

Have you ever been to a basketball game where they hold contests at halftime? A popular contest is one where the contestant needs to make a basket from half court to win the prize. Contestants often shoot the ball in different ways. They might take a regular basketball shot, a hook shot, or an underhand toss.

What's the best way to shoot the basketball to make a basket? In this video you'll see various shots and you decide!

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8-1 Quadratic Graphs and Their Properties

TEKS FOCUS

TEKS (6)(A) Determine the domain and range of quadratic functions and represent the domain and range using inequalities.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(B), (7)(A)

VOCABULARY

- **Axis of symmetry** – the line that divides a parabola into two matching halves
- **Falling object model** – The function $h = -16t^2 + c$ models the height of a falling object, where h is the object's height in feet, t is the time in seconds since the object began to fall, and c is the object's initial height.
- **Maximum** – The vertex of a parabola that opens downward is a maximum point. The y-coordinate of the vertex is the maximum value of the function.
- **Minimum** – The vertex of a parabola that opens upward is a minimum point. The y-coordinate of the vertex is the minimum value of the function.
- **Parabola** – the graph of a quadratic function
- **Quadratic function** – a function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$
- **Quadratic parent function** – the simplest quadratic function $f(x) = x^2$ or $y = x^2$
- **Standard form of a quadratic function** – $f(x) = ax^2 + bx + c$, where $a \neq 0$
- **Vertex** – the highest or lowest point on a parabola

- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

A quadratic function is a type of nonlinear function that models certain situations where the rate of change is not constant. The graph of a quadratic function is a symmetric curve with a highest or lowest point corresponding to a maximum or minimum value.

take note

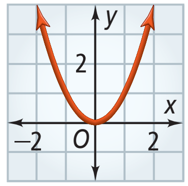
Key Concept Standard Form of a Quadratic Function

A **quadratic function** is a function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. This form is called the **standard form of a quadratic function**.

Examples $y = 3x^2$ $y = x^2 + 9$ $y = x^2 - x - 2$

take note

Key Concept Parabolas



The simplest quadratic function $f(x) = x^2$ or $y = x^2$ is the **quadratic parent function**.

The graph of a quadratic function is a U-shaped curve called a **parabola**. The parabola with equation $y = x^2$ is shown at the right.

You can fold a parabola so that the two sides match exactly. This property is called *symmetry*. The fold or line that divides the parabola into two matching halves is called the **axis of symmetry**.

The highest or lowest point of a parabola is its **vertex**, which is on the axis of symmetry. For graphs of functions of the form $y = ax^2$, the vertex is at the origin. The axis of symmetry is the y-axis, or $x = 0$.

If $a > 0$ in $y = ax^2 + bx + c$, the parabola opens upward.
↓
The vertex is the **minimum** point, or lowest point, of the parabola.

If $a < 0$ in $y = ax^2 + bx + c$, the parabola opens downward.
↓
The vertex is the **maximum** point, or highest point, of the parabola.

Problem 1

Identifying Key Attributes of a Quadratic Function

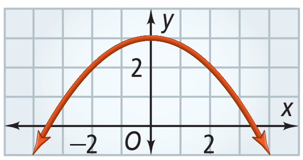
Graph each function. Identify the vertex and the axis of symmetry of the parabola. Tell whether the vertex is a minimum or maximum point, and identify the minimum or maximum value of the function. What is the y-intercept of the parabola?

Think

Can a parabola have both a minimum and a maximum point?
No. A parabola either opens upward and has a minimum point or opens downward and has a maximum point.

A $y = -\frac{1}{4}x^2 + 3$

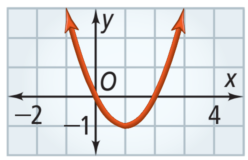
The function passes through the points $(-4, -1)$, $(-2, 2)$, $(0, 3)$, $(2, 2)$, and $(4, -1)$. Since $a = -\frac{1}{4}$ and $-\frac{1}{4} < 0$, the parabola opens downward.



The vertex is $(0, 3)$ and the axis of symmetry is the line $x = 0$. The vertex is a maximum point, and the maximum value of the function is 3, the y-coordinate of the vertex. The y-intercept is 3.

B $y = x^2 - 2x$

The function passes through the points $(-1, 3)$, $(0, 0)$, $(1, -1)$, $(2, 0)$, and $(3, 3)$. Since $a = 1$ and $1 > 0$, the parabola opens upward.



The vertex is $(1, -1)$ and the axis of symmetry is the line $x = 1$. The vertex is a minimum point, and the minimum value of the function is -1, the y-coordinate of the vertex. The y-intercept is 0.

Problem 2

TEKS Process Standard (1)(D)

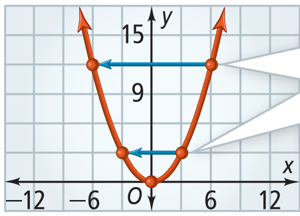
Graphing $y = ax^2$

Plan

What are good values to choose for x when making the table?
Choose values of x that make x^2 divisible by 3 so that the y-values will be integers.

Graph the function $y = \frac{1}{3}x^2$. Make a table of values. What are the domain and range?

x	$y = \frac{1}{3}x^2$	(x, y)
0	$\frac{1}{3}(0)^2 = 0$	(0, 0)
3	$\frac{1}{3}(3)^2 = 3$	(3, 3)
6	$\frac{1}{3}(6)^2 = 12$	(6, 12)



Reflect the points from the table over the axis of symmetry, $x = 0$, to find more points on the graph.

The domain is all real numbers. The range is $y \geq 0$.

Problem 3

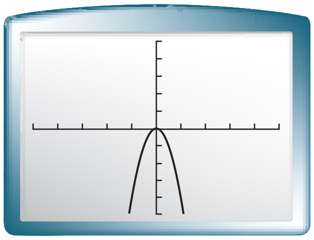
Comparing Widths of Parabolas

Think

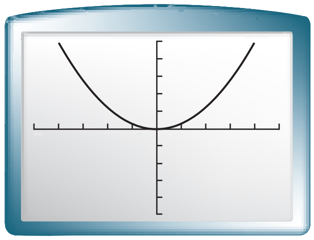
Does the sign of the x^2-term affect the parabola's width?
No. The sign of the x^2-term affects only whether the parabola opens upward or downward.

Use the graphs below. What is the order, from widest to narrowest, of the graphs of the quadratic functions $f(x) = -4x^2$, $f(x) = \frac{1}{4}x^2$, and $f(x) = x^2$?

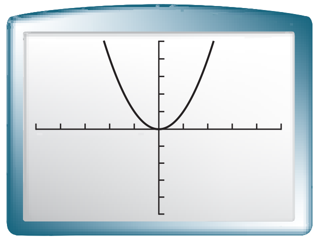
$f(x) = -4x^2$



$f(x) = \frac{1}{4}x^2$



$f(x) = x^2$



Of the three graphs, $f(x) = \frac{1}{4}x^2$ is the widest and $f(x) = -4x^2$ is the narrowest. So, the order from widest to narrowest is $f(x) = \frac{1}{4}x^2$, $f(x) = x^2$, and $f(x) = -4x^2$.

Problem 4

TEKS Process Standard (1)(D)

Graphing $y = ax^2 + c$

Multiple Choice **How is the graph of $y = 2x^2 + 3$ different from the graph of $y = 2x^2$?**

(A) It is shifted 3 units up.

(B) It is shifted 3 units down.

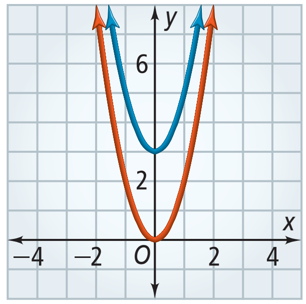
(C) It is shifted 3 units to the right.

(D) It is shifted 3 units to the left.

Plan

What values should you choose for *x*? Use the same values of *x* for graphing both functions so that you can see the relationship between corresponding *y*-coordinates.

x	$y = 2x^2$	$y = 2x^2 + 3$
−2	8	11
−1	2	5
0	0	3
1	2	5
2	8	11



The graph of $y = 2x^2 + 3$ has the same shape as the graph of $y = 2x^2$ but is shifted up 3 units. The correct answer is A.

Problem 5

TEKS Process Standard (1)(B)

Using the Falling Object Model

A **Nature** **An acorn drops from a tree branch 20 ft above the ground. The function $h = -16t^2 + 20$ gives the height h of the acorn (in feet) after t seconds. What is the graph of this quadratic function? At about what time does the acorn hit the ground?**

Know
- The function for the acorn's height
- The initial height is 20 ft.

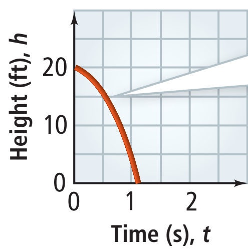
Need
The function's graph and the time the acorn hits the ground

Plan
Use a table of values to graph the function. Use the graph to estimate when the acorn hits the ground.

Think

Can you choose negative values for *t*? No. *t* represents time, so it cannot be negative.

t	$h = -16t^2 + 20$
0	20
0.5	16
1	4
1.5	−16



Graph the function using the first three ordered pairs from the table. Do not plot (1.5, −16) because height cannot be negative.

The acorn hits the ground when its height above the ground is 0 ft, or when $h = 0$. By estimating the t-intercept of the graph, you can see that the acorn hits the ground after about 1.1 s.

continued on next page ▶

Problem 5 *continued*

B **What are a reasonable domain and range for the function $h = -16t^2 + 20$ described in part (A)?**

Time t and height h cannot be negative. You can use the graph in part (A) to determine the domain and range of the function $h = -16t^2 + 20$. Note that the maximum value of t, 1.1, is an approximate value.

Domain: $0 \le t \le 1.1$ Range: $0 \le h \le 20$

PRACTICE and APPLICATION EXERCISES

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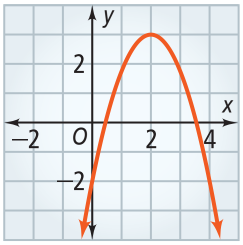
Identify the vertex and the axis of symmetry of each parabola. Tell whether the vertex is a minimum or a maximum, and identify the minimum or maximum value of the function.

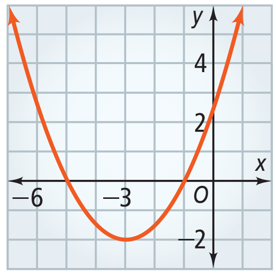
1.



2.

x	y
0	8
1	2
2	0
3	2
4	8

3.

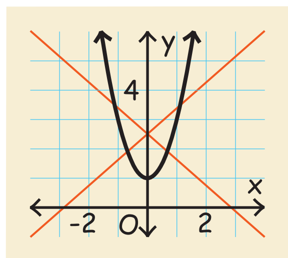


Graph each function. Then identify the domain and range of the function.

4. $y = -4x^2$ **5.** $f(x) = 1.5x^2$ **6.** $f(x) = 3x^2$

7. **Use a Problem Solving Model (1)(B)** A person walking across a bridge accidentally drops an orange into the river below from a height of 40 ft. The function $h = -16t^2 + 40$ gives the orange's approximate height h above the water, in feet, after t seconds. Graph the function. In how many seconds will the orange hit the water? Use inequalities to describe a reasonable domain and range for the function.

8. **Explain Mathematical Ideas (1)(G)** Describe and correct the error made in graphing the function $y = -2x^2 + 1$.



9. **Use a Problem Solving Model (1)(B)** A bird drops a stick to the ground from a height of 80 ft. The function $h = -16t^2 + 80$ gives the stick's approximate height h above the ground, in feet, after t seconds. Graph the function. At about what time does the stick hit the ground? Use inequalities to describe a reasonable domain and range for the function.

Identify the domain and range of each function.

10. $f(x) = 3x^2 + 6$ **11.** $y = -2x^2 - 1$

12. $y = -\frac{3}{4}x^2 - 9$ **13.** $y = \frac{2}{3}x^2 + 12$

14. What information do the numbers a and c give you about the graph of $y = ax^2 + c$?

Order each group of quadratic functions from widest to narrowest graph.

15. $y = 3x^2, y = 2x^2, y = 4x^2$

16. $y = -\frac{1}{2}x^2, y = 5x^2, y = -\frac{1}{4}x^2$

17. **Apply Mathematics (1)(A)** Suppose a person is riding in a hot-air balloon, 154 ft above the ground. He drops an apple. The height h, in feet, of the apple above the ground is given by the formula $h = -16t^2 + 154$, where t is the time in seconds. To the nearest tenth of a second, at what time does the apple hit the ground?

Use Multiple Representations to Communicate Mathematical Ideas (1)(D) **Graph each function. Identify the vertex and the axis of symmetry of the parabola. Tell whether the vertex is a minimum or a maximum point, and identify the minimum or maximum value of the function. Identify the y-intercept of the parabola.**

18. $f(x) = x^2 + 4$

19. $y = x^2 - 7$

20. $y = \frac{1}{2}x^2 + 2$

21. $f(x) = -x^2 - 3$

22. $y = -2x^2 + 4$

23. $f(x) = 4x^2 - 5$

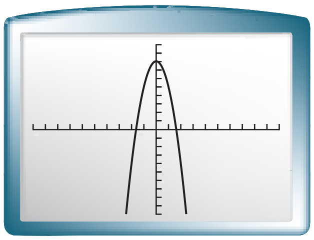
Match each function with its graph.

24. $f(x) = x^2 - 1$

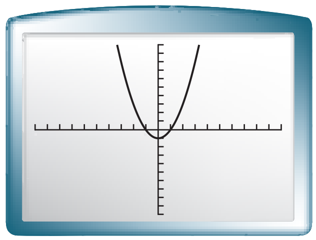
25. $f(x) = -3x^2 + 8$

26. $f(x) = -0.2x^2 + 5$

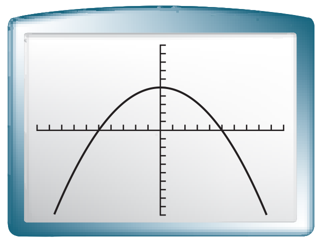
A.



B.



C.



27. Using a graphing calculator, graph $f(x) = x^2 + 2$.

a. If $f(x) = x^2 + 2$ and $g(x) = 3f(x)$, write the equation for $g(x)$. Graph $g(x)$ and compare the graph to the graph of $f(x)$.

b. If $f(x) = x^2 + 2$ and $h(x) = f(3x)$, write the equation for $h(x)$. Graph $h(x)$ and compare the graph to the graph of $f(x)$.

c. Compare how multiplying a quadratic function by a number and multiplying the x-value of a quadratic function by a number change the graphs of the quadratic functions.

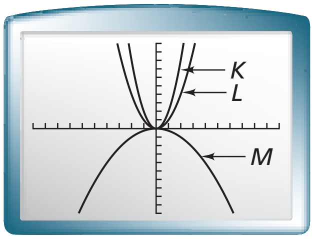
Use a graphing calculator to graph each function. Identify the vertex and axis of symmetry.

28. $y = \frac{1}{4}x^2 + 3$

29. $f(x) = -1.5x^2 + 5$

30. $y = -3x^2 - 6$

Three graphs are shown at the right. Identify the graph or graphs that fit each description.



31. $a > 0$

32. $a < 0$

33. $|a|$ has the greatest value.

34. $|a|$ has the least value.

35. Consider the graphs of $y = ax^2$ and $y = (ax)^2$. Assume $a \neq 0$.

a. For what values of a will both graphs lie in the same quadrant(s)?

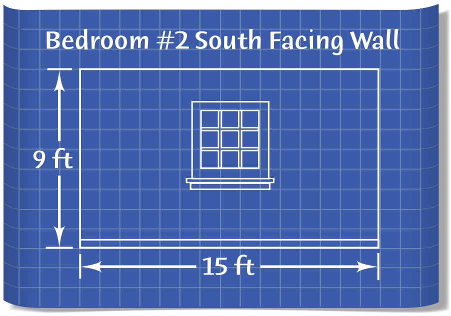
b. For what values of a will the graph of $y = ax^2$ be wider than the graph of $y = (ax)^2$?

36. Complete each statement. Assume $a \neq 0$.

a. The graph of $y = ax^2 + c$ intersects the x-axis in two places when _?_.

b. The graph of $y = ax^2 + c$ does not intersect the x-axis when _?_.

STEM **37. Apply Mathematics (1)(A)** A blueprint is given for a rectangular wall with a square window in the center. If each side of the window is x feet, the function $y = 135 - x^2$ gives the area (in square feet) of the wall without the window.



a. Graph the function.

b. What is a reasonable domain for the function? Explain.

c. What is the range of the function? Explain.

d. Estimate the side length of the window if the area of the wall is 117 ft^2.

TEXAS End-of-Course PRACTICE

38. Which equation has a graph that is narrower than the graph of $y = 4x^2 + 5$?

A. $y = 4x^2 - 5$ **B.** $y = -5x^2 + 4$ **C.** $y = 0.75x^2 + 5$ **D.** $y = -0.75x^2 - 4$

39. Kristina is evaluating some formulas as part of a science experiment. One of the formulas involves the expression $24 - (-17)$. What is the value of this expression?

F. -41 **G.** -7 **H.** 7 **J.** 41

40. Which expression is equivalent to $8(x + 9)$?

A. $x + 72$ **B.** $8x + 72$ **C.** $8x + 17$ **D.** $8x + 9$

41. What is the solution of the equation $2(x + 3) + 7 = -11$?

F. -12 **G.** -1 **H.** 1 **J.** 12

42. A rectangular dog run has an area of $x^2 - 22x - 48$. What are possible dimensions of the dog run? Use factoring. Explain how you found the dimensions.

8-2 Quadratic Functions

TEKS FOCUS

TEKS (7)(A) Graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(A), (1)(C), (8)(B)

VOCABULARY

- **Vertical motion model** – The function $h = -16t^2 + vt + c$ models the height *h* of an object, in feet, *t* seconds after it is projected into the air from initial height *c*, in feet, with an initial upward velocity *v*, in feet per second.
- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

In the quadratic function $y = ax^2 + bx + c$, the value of *b* affects the position of the axis of symmetry.

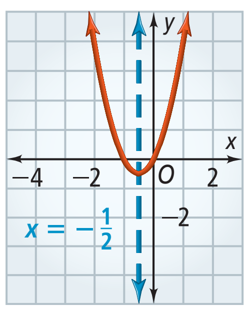
take note

Key Concept Axis of Symmetry of a Quadratic Function

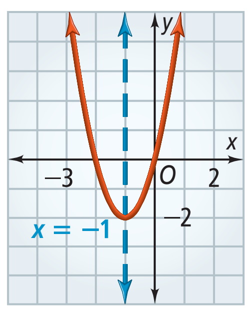
The graph of $y = ax^2 + bx + c$, where $a \neq 0$, has the line $x = \frac{-b}{2a}$ as its axis of symmetry. The *x*-coordinate of the vertex is $\frac{-b}{2a}$.

The examples below show how the position of the axis of symmetry changes as the *b*-value changes. In each case, the equation of the axis of symmetry is $x = \frac{-b}{2a}$.

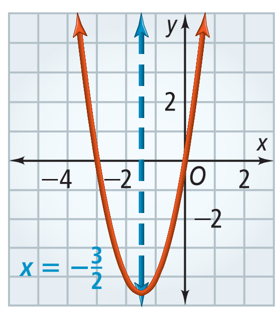
$y = 2x^2 + 2x$



$y = 2x^2 + 4x$



$y = 2x^2 + 6x$



axis of symmetry: $x = -\frac{1}{2}$ $x = -1$ $x = -\frac{3}{2}$

Problem 1

TEKS Process Standard (1)(D)

Graphing $y = ax^2 + bx + c$

What is the graph of the function $y = x^2 - 6x + 4$?

Think

How are the vertex and the axis of symmetry related?
The vertex is on the axis of symmetry. You can use the equation for the axis of symmetry to find the x-coordinate of the vertex.

Step 1 Find the axis of symmetry and the coordinates of the vertex.

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$$ Find the equation of the axis of symmetry.

The axis of symmetry is $x = 3$. So the x-coordinate of the vertex is 3.

$y = x^2 - 6x + 4$

$= 3^2 - 6(3) + 4$ Substitute 3 for x to find the y-coordinate of the vertex.

$= -5$ Simplify.

The vertex is $(3, -5)$.

Step 2 Find two other points on the graph.

Find the y-intercept. When $x = 0$, $y = 4$, so one point is $(0, 4)$.

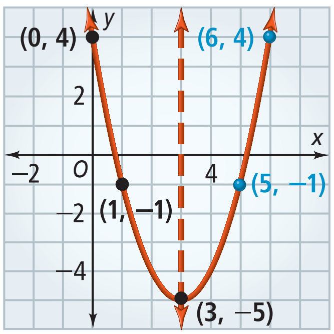
Find another point by choosing a value for x on the same side of the vertex as the y-intercept. Let $x = 1$.

$y = x^2 - 6x + 4$

$= 1^2 - 6(1) + 4 = -1$ Substitute 1 for x and simplify.

When $x = 1$, $y = -1$, so another point is $(1, -1)$.

Step 3 Graph the vertex and the points you found in Step 2, $(0, 4)$ and $(1, -1)$. Reflect the points from Step 2 across the axis of symmetry to get two more points on the graph. Then connect the points with a parabola.

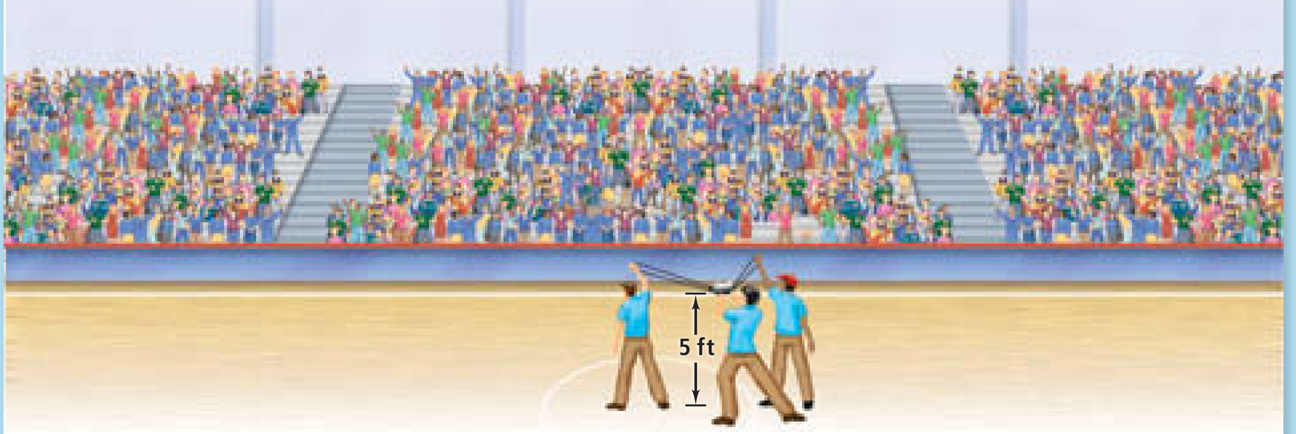


Problem 2

TEKS Process Standard (1)(C)

Using the Vertical Motion Model

Entertainment **During halftime of a basketball game, a slingshot launches T-shirts at the crowd. A T-shirt is launched with an initial upward velocity of 72 ft/s. The T-shirt is caught 35 ft above the court. How long will it take the T-shirt to reach its maximum height? What is its maximum height? What is the range of the function that models the height of the T-shirt over time?**



To solve this problem, use the vertical motion model $h = -16t^2 + vt + c$, where h is the height of an object, in feet, t seconds after it is projected into the air from initial height c, in feet, with initial upward velocity v, in feet per second.

Plan

What are the values of v and c?
The T-shirt is launched from a height of 5 ft, so $c = 5$. The T-shirt has an initial upward velocity of 72 ft/s, so $v = 72$.

The function $h = -16t^2 + 72t + 5$ gives the T-shirt's height h, in feet, after t seconds. Since the coefficient of t^2 is negative, the parabola opens downward, and the vertex is the maximum point.

Method 1 Use paper and pencil with a formula.

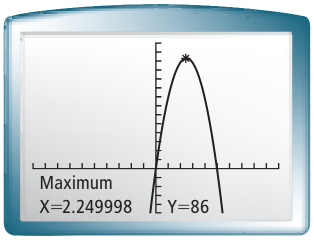
$$t = \frac{-b}{2a} = \frac{-72}{2(-16)} = \frac{9}{4}$$ Find the t-coordinate of the vertex.

$$h = -16\left(\frac{9}{4}\right)^2 + 72\left(\frac{9}{4}\right) + 5 = 86$$ Find the h-coordinate of the vertex.

The T-shirt will reach its maximum height of 86 ft after $\frac{9}{4}$ s, or 2.25 s. The range describes the height of the T-shirt during its flight. The T-shirt starts at 5 ft, peaks at 86 ft, and then is caught at 35 ft. The height of the T-shirt at any time is between 5 ft and 86 ft, inclusive, so the range is $5 \le h \le 86$.

Method 2 Use technology.

Enter the function $h = -16t^2 + 72t + 5$ as $y = -16x^2 + 72x + 5$ on the **Y =** screen and graph the function. Use the **CALC** feature and select **MAXIMUM**. Set left and right bounds on the maximum point and calculate the point's coordinates. The coordinates of the maximum point are (2.25, 86).



The T-shirt will reach its maximum height of 86 ft after 2.25 s. The range of the function is $5 \le h \le 86$.

Problem 3

Fitting a Quadratic Function to Data

The table shows the run times, in seconds, a certain computer program requires to analyze the given megabytes of data.

Data (MB)	1	2	3	4	5
Run Time (s)	1.3	3.3	6.7	10.7	16

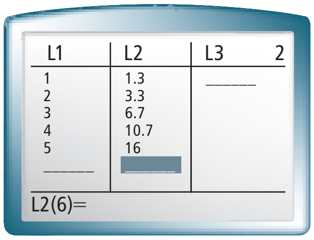
Think

Will a quadratic function with all integer coefficients fit the data exactly? No. The table shows that for integer x-values 1 to 5, the y-values are *not* integers. This means that at least one of the coefficients a, b, and c is not an integer.

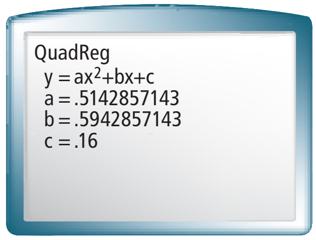
A **Find a quadratic function $f(x) = ax^2 + bx + c$ that provides a reasonable fit to the data.**

Use technology to find a, b, and c. Let x represents the data size in megabytes and let y represent the run time in seconds.

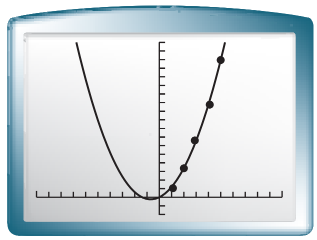
Enter the data into lists.



Find the quadratic regression.



A quadratic function that models the data is $f(x) = 0.51x^2 + 0.59x + 0.16$. Graphing the function with the data shows that the quadratic function provides a reasonable fit to the data, so the function can be used to estimate solutions to problems or to make predictions.



B **Estimate the run time to analyze 3.5 MB of data.**

The variable x represents megabytes of data. Evaluate $f(3.5)$.

$f(x) = 0.51x^2 + .59x + 0.16$	Write the quadratic function.
$f(3.5) = 0.51(3.5)^2 + .59(3.5) + 0.16$	Substitute 3.5 for x.
≈ 8.47	Simplify.

The run time to analyze 3.5 MB of data is about 8.5 seconds.

C **Predict the run time for 10 MB of data.**

Evaluate $f(10)$.

$f(x) = 0.51x^2 + 0.59x + 0.16$	Write the quadratic function.
$f(10) = 0.51(10)^2 + 0.59(10) + 0.16$	Substitute 10 for x.
$= 57.06$	Simplify.

The model predicts that the computer program will need to run for about 57.1 seconds to analyze 10 MB of data.

PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to PearsonTEXAS.com.

Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of each function.

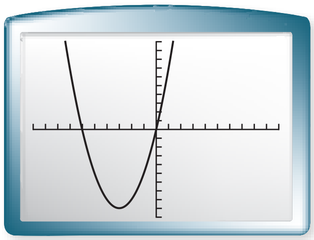
1. $y = 2x^2 + 3$ **2.** $y = -3x^2 + 12x + 1$ **3.** $f(x) = 2x^2 + 4x - 1$

4. $y = x^2 - 8x - 7$ **5.** $f(x) = 3x^2 - 9x + 2$ **6.** $y = -4x^2 + 11$

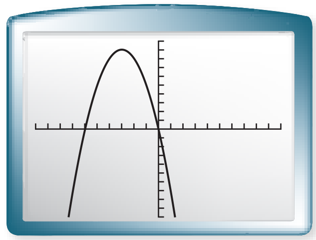
Match each function with its graph.

7. $y = -x^2 - 6x$ **8.** $y = -x^2 + 6$ **9.** $y = x^2 - 6$ **10.** $y = x^2 + 6x$

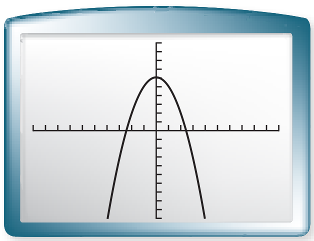
A.



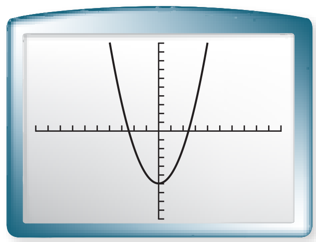
C.



B.



D.



11. **Select Tools to Solve Problems (1)(C)** A baseball is thrown into the air with an upward velocity of 30 ft/s. Its height h, in feet, after t seconds is given by the function $h = -16t^2 + 30t + 6$. How long will it take the ball to reach its maximum height? What is the ball's maximum height? What is the range of the function? Select a tool, such as paper and pencil or technology, to solve this problem.

12. **Apply Mathematics (1)(A)** Suppose you have 100 ft of string to rope off a rectangular section for a bake sale at a school fair. The function $A = -x^2 + 50x$ gives the area of the section in square feet, where x is the width in feet. What width gives you the maximum area you can rope off? What is the maximum area? What is the range of the function?

Use Multiple Representations to Communicate Mathematical Ideas (1)(D)
Use the axis of symmetry, vertex, and y-intercept to graph each function. Label the axis of symmetry and the vertex.

13. $f(x) = x^2 + 4x - 5$ **14.** $y = 2x^2 - 6x + 1$ **15.** $y = -2x^2 + 8x + 9$

16. **Explain Mathematical Ideas (1)(G)** Describe and correct the error made in finding the axis of symmetry for the graph of $y = -x^2 - 6x + 2$.

$x = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$

17. **a.** What is the vertex of the function $y = 5x^2 + 10x + 24$?

b. What is the vertex of the function given in the table?

x	−4	−3	−2	−1	0
y	3	−3	−5	−3	3

18. **Apply Mathematics (1)(A)** The Riverside Geyser in Yellowstone National Park erupts about every 6.25 h. When the geyser erupts, the water has an initial upward velocity of 69 ft/s. What is the maximum height of the geyser? Round your answer to the nearest foot.

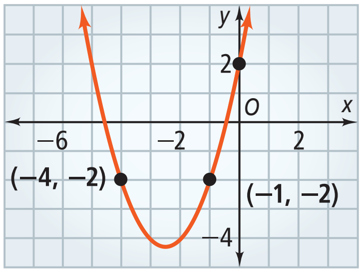
19. **Apply Mathematics (1)(A)** A cellphone company sells about 500 phones each week when it charges \$75 per phone. It sells about 20 more phones per week for each \$1 decrease in price. The company's revenue is the product of the number of phones sold and the price of each phone. What price should the company charge to maximize its revenue?

20. **Apply Mathematics (1)(A)** Suppose a tennis player hits a ball over the net. The ball leaves the racket 0.5 m above the ground. The equation $h = -4.9t^2 + 3.8t + 0.5$ gives the ball's height h in meters after t seconds.

a. When will the ball be at the highest point in its path? Round to the nearest tenth of a second.

b. If you double your answer from part (a), will you find the amount of time the ball is in the air before it hits the court? Explain.

21. The parabola at the right is of the form $y = x^2 + bx + c$.



a. Use the graph to find the y-intercept.

b. Use the graph to find the equation of the axis of symmetry.

c. Use the formula $x = \frac{-b}{2a}$ to find b.

d. Write the equation of the parabola.

e. Test one point using your equation from part (d).

f. Would this method work if the value of a were not known? Explain.

22. The table shows data a biologist gathered about the population of a bacteria colony. Use technology to find a quadratic function that is a reasonable fit to the data. Use the function to estimate the bacteria population at 12 days. Round to the nearest million bacteria.

Time (days)	0	1	2	3	4	5	6
Population (millions)	9	7	3	4	8	10	19

23. **Select Tools to Solve Problems (1)(C)** A laser tracks the flight of a hit baseball. The table shows the height of the ball in feet as a function of time in seconds. Use technology to find a quadratic function that is a reasonable fit to the data. Use the function to predict the height of the ball at 4.5 seconds. Round to the nearest hundredth of a foot.

Time (s)	0	0.5	1.0	1.5	2.0	2.5
Height (ft)	4	37.5	63	80.5	90	91.5

24. **Select Tools to Solve Problems (1)(C)** The table shows examples of the expected return on an investment in a new company. Use technology to find a quadratic function that is a reasonable fit to the data. Use the function to predict the return on an investment of \$4000. Round to the nearest thousand dollars.

Investment (\$)	50	100	200	500	1000	1500
Return (\$)	2	12	50	250	1000	2000

25. A meteorologist records the temperature data that is shown in the table. Does the data suggest that there is a quadratic model that is a reasonable fit? Justify your answer.

Number of Days	0	6	12	18	24	30	36	42	48
Temperature (°F)	41	38	37	46	51	52	47	44	41

TEXAS End-of-Course PRACTICE

26. A half-pipe ramp at a skate park is approximately parabolic in shape. It can be modeled by the quadratic function $y = x^2 - 6x + 9$. At what point would a skater be at the lowest part of the ramp?

A. $(-3, 36)$ **B.** $(36, -3)$ **C.** $(3, 0)$ **D.** $(0, 3)$

27. What is the simplified form of the product $4(-8)(5)(-1)$?

F. -160 **G.** -80 **H.** 80 **J.** 160

28. Which of the following is equivalent to $(-4)^3$?

A. -64 **B.** -12 **C.** 12 **D.** 64

29. Toby needs to write an example of the Commutative Property of Multiplication for his homework. Which of the following expressions could he use?

F. $ab = ba$ **G.** $a = a$ **H.** $ab = ab$ **J.** $a(bc) = (ab)c$

30. Simplify the product $(3r - 1)(4r^2 + r + 2)$. Justify each step.

8-3 Transformations of Quadratic Functions

TEKS FOCUS

TEKS (7)(C) Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(A), (1)(F), (7)(A)

VOCABULARY

- **Compression** – A compression is a transformation that decreases the distance between corresponding points of a graph and a line.
- **Stretch** – A stretch is a transformation that increases the distance between corresponding points of a graph and a line.

- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

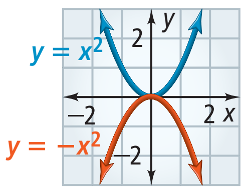
The graph of any quadratic function is a transformation of the graph of the parent quadratic function, $y = x^2$.

take note

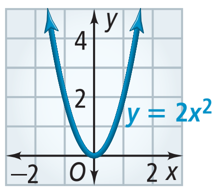
Key Concept Reflection, Stretch, and Compression

Vertical reflection, stretch, and compression

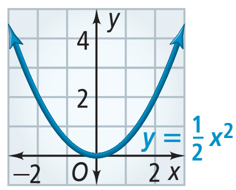
The graphs of $y = ax^2$ and $y = -ax^2$ are reflections of each other across the x-axis.
When $|a| > 1$, the graph of $y = x^2$ is stretched vertically.
When $|a| < 1$, the graph of $y = x^2$ is compressed vertically.



Reflection, a and $-a$



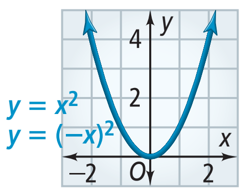
Stretch, $a > 1$



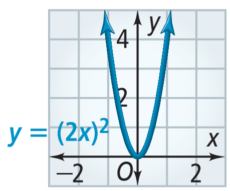
Compression, $0 < a < 1$

Horizontal reflection, stretch, and compression

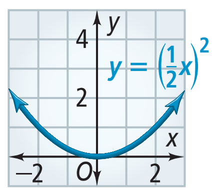
The graphs of $y = (bx)^2$ and $y = (-bx)^2$ are reflections of each other across the y-axis.
When $|b| > 1$, the graph of $y = x^2$ is compressed horizontally.
When $|b| < 1$, the graph of $y = x^2$ is stretched horizontally.



Reflection, b and $-b$



Compression, $b > 1$



Stretch, $0 < b < 1$

Key Concept Translation

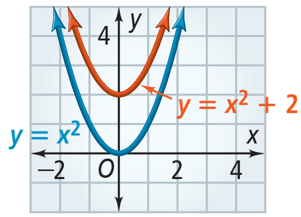
take note

Vertical translation

The graph of $y = x^2 + d$ is a vertical translation of the parent function $y = x^2$.

When $d > 0$, the graph of $y = x^2$ shifts d units up.

When $d < 0$, the graph of $y = x^2$ shifts $|d|$ units down.

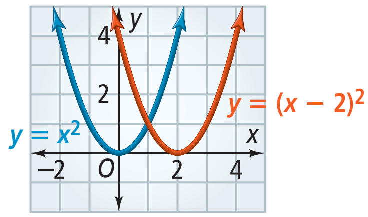


Horizontal translation

The graph of $y = (x - c)^2$ is a horizontal translation of the parent function $y = x^2$.

When $c > 0$, the graph of $y = x^2$ shifts c units right.

When $c < 0$, the graph of $y = x^2$ shifts $|c|$ units left.



Problem 1

Graphing Quadratic Functions With Stretch and Compression

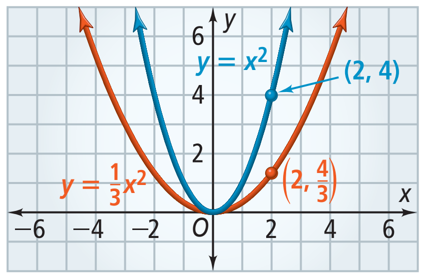
Think

How can you use the new equation to identify the change to the graph of the parent function? Think about how the change in the equation affects the coordinates of a particular point, such as (2, 4). In part (A), the point (2, 4) becomes the point $\left(2, \frac{4}{3}\right)$.

Graph each function. How is each graph a stretch or compression of $f(x) = x^2$?

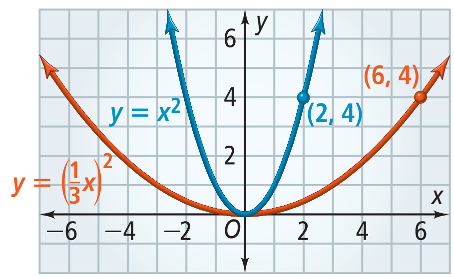
A $y = \frac{1}{3}x^2$

Graph the parent function, $f(x) = x^2$. The y-coordinate of each point on the graph of $y = \frac{1}{3}x^2$ is one-third the y-coordinate of the corresponding point on the graph of the parent function. The graph of $y = \frac{1}{3}x^2$ is a vertical compression of the graph of $f(x) = x^2$.



B $y = \left(\frac{1}{3}x\right)^2$

Graph the parent function, $f(x) = x^2$. The x-coordinate of each point on the graph of $y = \left(\frac{1}{3}x\right)^2$ is 3 times the x-coordinate of the corresponding point on the graph of the parent function. The graph of $y = \left(\frac{1}{3}x\right)^2$ is a horizontal stretch of the graph of $f(x) = x^2$.



Problem 2 TEKS Process Standard (1)(D)

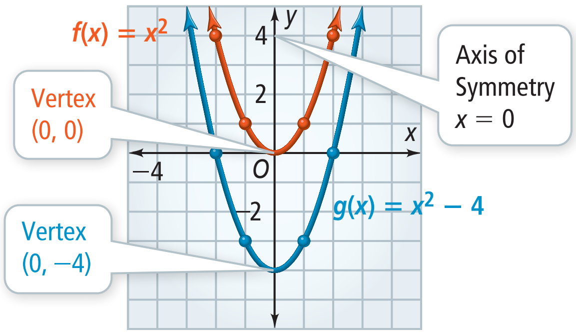
Graphing Translations of $f(x) = x^2$

Graph each function. How is each graph a translation of $f(x) = x^2$?

Think

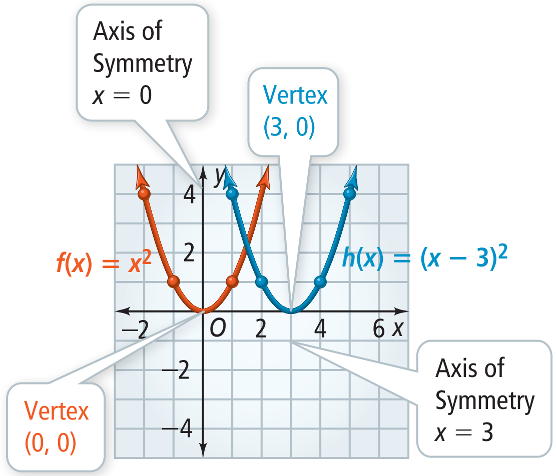
How does $g(x)$ differ from $f(x)$?
For each value of x, the value of $g(x)$ is 4 less than the value of $f(x)$.

A $g(x) = x^2 - 4$



Translate the graph of $f(x)$ down 4 units to get the graph of $g(x) = x^2 - 4$.

B $h(x) = (x - 3)^2$



Translate the graph of $f(x)$ to the right 3 units to get the graph of $h(x) = (x - 3)^2$.

Think

What does the value of a tell you?
The value of a tells you whether the graph of the parent function, $y = x^2$, is stretched or compressed vertically. If a is negative, the graph is a reflection across the x-axis.

Problem 3

TEKS Process Standard (1)(F)

Describing a Combination of Transformations

Multiple Choice What steps transform the graph of $y = x^2$ to $y = -3(x + 4)^2 + 2$?

(A) Reflect across the x-axis, stretch by the factor 3, translate 4 units to the right and 2 units up.

(B) Stretch by the factor 3, translate 4 units to the right and 2 units up.

(C) Reflect across the x-axis, translate 4 units to the left and 2 units up.

(D) Stretch by the factor 3, reflect across the x-axis, translate 4 units to the left and 2 units up.

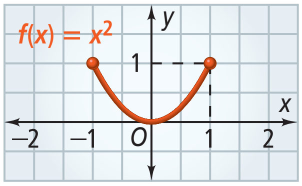
$|a| > 1$ and a is negative, so the graph is stretched vertically and reflects across the x-axis. Since $x + 4 = x - (-4)$, you know that $c < 0$, so the graph shifts $|c|$ units to the left. $d > 0$, so the graph shifts d units up. The correct choice is D.

Problem 4

TEKS Process Standard (1)(A)

Applying Transformations of Quadratic Functions

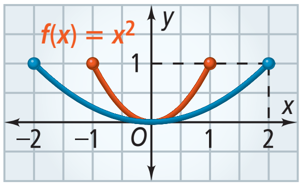
A company makes satellite dishes with cross sections that can be modeled by quadratic functions. The parent function $f(x) = x^2$ models a cross section of a satellite dish that is 1 ft deep and has a radius of 1 ft. On the graph, the radius of the satellite dish is the distance of 1 ft from the y-axis and the depth is the distance of 1 ft from the x-axis.



The company needs to make a new satellite dish that has a cross section 1 ft deep with a radius of 2 ft. What is a quadratic function that models the cross section of the new dish?

Draw a graph of the parent function and the new function. The new dish has the same vertex as the parent function, (0, 0), but is stretched 2 times as wide. So the equation is in the form $y = (bx)^2$, because the graph has been stretched horizontally. A horizontal stretch means that $|b| < 1$. A horizontal reflection will not change the graph because the axis of symmetry is the y-axis, so you can choose $b > 0$. Since the new graph is twice as wide and $b < 1$, $b = \frac{1}{2}$.

A quadratic function modeling the cross section is $y = \left(\frac{1}{2}x\right)^2$, or $y = \frac{1}{4}x^2$.



PRACTICE and APPLICATION EXERCISES

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In Exercises 1–8, $f(x) = x^2$. Write an equation of the function $g(x)$ and describe the effects on the graph of the parent function $f(x) = x^2$.

1. $g(x) = f(x) + d$, where $d = -1$

2. $g(x) = f(x - c)$, where $c = -3$

3. $g(x) = af(x)$, where $a = -1$

4. $g(x) = af(x)$, where $a = 0.4$

5. $g(x) = f(bx)$, where $b = 3$

6. $g(x) = f(bx)$, where $b = -1$

7. $g(x) = f(x) + d$, where $d = 2$

8. $g(x) = f(x - c)$, where $c = 4$

Use Representations to Communicate Mathematical Ideas (1)(E)

Sketch the graph of the function. Identify the vertex and axis of symmetry.

9. $f(x) = x^2$

10. $f(x) = -\frac{1}{8}x^2 + 8$

11. $f(x) = (x + 3)^2 - 2$

12. $f(x) = 3(x + 1)^2 - 4$

13. $f(x) = -(x - 1.5)^2 + 5$

14. $f(x) = \frac{1}{2}(x - 3)^2$

Sketch the graphs of each pair of functions on the same coordinate grid. Identify the vertex of each graph. If the second graph is a transformation of the first, identify the transformation as a stretch, compression, reflection, or translation.

15. $f(x) = x^2$, $g(x) = 2x^2$

16. $f(x) = -\frac{1}{3}x^2$; $g(x) = -\frac{1}{3}x^2 - 5$

17. $f(x) = x^2$, $g(x) = (x - 3)^2 - 1$

18. $f(x) = (x + 2)^2$; $g(x) = (2x + 4)^2$

19. $f(x) = (3x)^2 + 1$; $g(x) = (-3x)^2 + 1$

20. $f(x) = \frac{1}{2}(x + 4)^2$; $g(x) = -\frac{1}{2}(x + 4)^2$

21. Analyze Mathematical Relationships (1)(F) When a ball is tossed into the air, its height is a quadratic function of time. One student models the height of the ball as $h(t) = -16t^2$. Another student models it as $h(t) = -16t^2 + 10$. Could both models be correct? Explain.

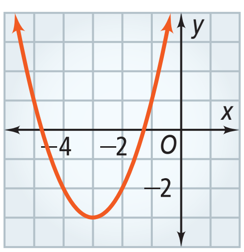
Match the function to the graph.

22. $y = 3x^2 - 3$

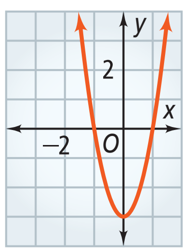
23. $y = -(x - 3)^2 + 3$

24. $y = (x + 3)^2 - 3$

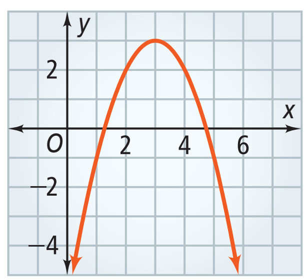
A.



B.



C.

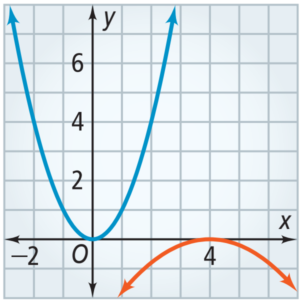


25. Explain Mathematical Ideas (1)(G) Let a be a nonzero real number such that $|a| \neq 1$. The graphs of $f(x) = x^2$ and $g(x) = (ax)^2$ have the same vertex of (0, 0). Do the graphs share any other points in common? Explain your reasoning.

26. Evaluate Reasonableness (1)(B) An engineer is designing a section of a roller coaster. The section runs 20 meters along the ground, and its shape is described by a quadratic function. In each function below, x represents the distance in meters along the ground, and y is the height of the track in meters. Which of the functions describes the most reasonable track? Explain your choice.

$y = -(x - 10)^2 + 80$ $\quad$ $y = -\frac{1}{4}(x - 10)^2 + 28$ $\quad$ $y = 5(x - 10)^2 + 28$

TEXAS End-of-Course PRACTICE



27. The graph shows two parabolas, one of which is described by $f(x) = x^2$. Which function describes the other parabola?

A. $y = -2(x - 4)^2 + 4$

B. $y = (2x)^2 - 4$

C. $y = 0.2(x + 4)^2$

D. $y = -0.2(x - 4)^2$

28. Which transformation maps the graph of $f(x) = x^2$ to the graph of $g(x) = (x + 4)^2$?

F. a reflection across the line $x = -4$

G. a reflection across the line $y = -4$

H. a translation shifting $f(x)$ 4 units to the left

J. a translation shifting $f(x)$ 4 units to the right

29. Which describes the transformation of the graph of $f(x) = x^2 + 2$ to the graph of $g(x) = (-3x)^2 + 2$?

A. a horizontal compression without reflection

B. a horizontal stretch without reflection

C. a vertical compression without reflection

D. a vertical stretch and a translation

30. Let n be a nonzero real number. Of the functions $y = x^2 + n$, $y = (x - n)^2$, and $y = (nx)^2$, which has the same axis of symmetry as the parent function $y = x^2$? Explain.

8-4 Vertex Form of a Quadratic Function

TEKS FOCUS

TEKS (6)(B) Write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$).

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (6)(A), (7)(A), (12)(B)

VOCABULARY

- **Vertex form** – The vertex form of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$ and (h, k) is the vertex of the function.
- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated

ESSENTIAL UNDERSTANDING

A quadratic function written in vertex form clearly indicates the vertex and axis of symmetry of the parabola, and the maximum or minimum value of the function.

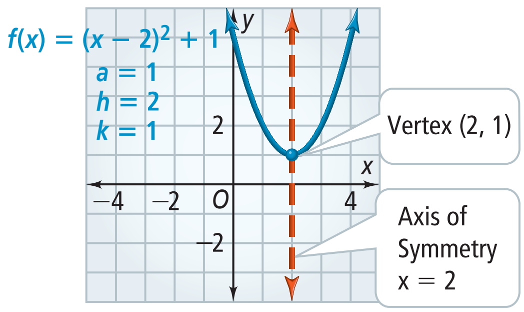
take note — Key Concept Vertex Form of a Quadratic Function

The quadratic function $f(x) = a(x - h)^2 + k$ is written in **vertex form**.

The line $x = h$ is the axis of symmetry of the parabola.

The vertex of the parabola is (h, k).

The maximum or minimum value of the function is k.



Problem 1

Plan

How do you use vertex form? Compare $y = 6(x - 5)^2 - 1$ to vertex form $y = a(x - h)^2 + k$ to find values for a, h, and k.

Interpreting Vertex Form

For $y = 6(x - 5)^2 - 1$, what are the vertex, the axis of symmetry, the maximum or minimum value, and the domain and range?

Step 1 Compare: $y = 6(x - 5)^2 - 1$
$y = a(x - h)^2 + k$

Step 2 The vertex is $(h, k) = (5, -1)$.

Step 3 The axis of symmetry is $x = h$, or $x = 5$.

Step 4 Since $a > 0$, the parabola opens upward. $k = -1$ is the minimum value.

Step 5 There is no restriction on the value of x, so the domain is all real numbers. Since the minimum value of the function is -1, the range is $y \geq -1$.

Problem 2

TEKS Process Standard (1)(D)

Using Vertex Form

What is the graph of $f(x) = -3(x - 1)^2 + 4$?

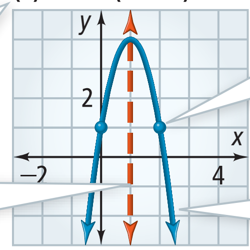
Think

How can you use the values of a, h, and k to check your graph? Check that the graph is a stretched reflection of $y = x^2$, shifted 1 unit right and 4 units up.

Step 1 Identify the constants $a = -3$, $h = 1$, and $k = 4$. Because $a < 0$, the parabola opens downward.

Step 2 Plot the vertex $(h, k) = (1, 4)$ and draw the axis of symmetry $x = 1$.

$f(x) = -3(x - 1)^2 + 4$



Step 3 Plot two points. $f(2) = -3(2 - 1)^2 + 4 = 1$. Plot (2, 1) and the reflected point (0, 1).

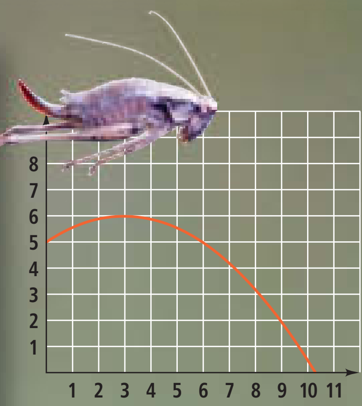
Step 4 Sketch the curve.

Problem 3

Writing a Quadratic Function in Vertex Form and in Standard Form

A **Nature** **The picture shows the jump of a cricket. What quadratic function models the path of the cricket's jump? Write the function in vertex form.**

Think	Write
What is the vertex? Use the vertex to identify h and k.	The vertex is (3, 6). $h = 3$, $k = 6$
Choose another point, (6, 5), from the path. Substitute for h, k, x, and $f(x)$ in the vertex form. Solve for a.	$f(x) = a(x - h)^2 + k$ $5 = a(6 - 3)^2 + 6$ $5 = 9a + 6$ $-1 = 9a$ $a = -\frac{1}{9}$
Substitute for a, h, and k in the vertex form.	$f(x) = -\frac{1}{9}(x - 3)^2 + 6$ models the path of the cricket's jump.



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Problem 3 *continued*

B Write the function in standard form.

$f(x) = -\frac{1}{9}(x-3)^2 + 6$ — Write the equation in vertex form.

$f(x) = -\frac{1}{9}(x^2 - 6x + 9) + 6$ — Square the binomial.

$f(x) = (-\frac{1}{9}x^2 + \frac{2}{3}x - 1) + 6$ — Use the Distributive Property.

$f(x) = -\frac{1}{9}x^2 + \frac{2}{3}x + 5$ — Simplify.

The function in standard form is $f(x) = -\frac{1}{9}x^2 + \frac{2}{3}x + 5$.

PRACTICE and APPLICATION EXERCISES

ONLINE HOMEWORK

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For each quadratic function in vertex form, identify the vertex of the parabola, the axis of symmetry, whether the parabola opens upward or downward, and the domain and range of the function.

1. $f(x) = 2(x-3)^2 - 10$

2. $f(x) = -\frac{1}{3}(x+6)^2 + 1$

3. $f(x) = \frac{2}{3}(x - \frac{4}{5})^2 - 9$

4. $f(x) = -12(x-1)^2 - \frac{1}{6}$

5. $f(x) = -1.5x^2 + 3.5$

6. $f(x) = 2(x+11)^2$

Sketch the graph of each of these functions. Label the vertex.

7. $f(x) = (x-2)^2 - 3$

8. $f(x) = -2(x+4)^2 + 6$

9. $f(x) = \frac{1}{2}(x-3.5)^2 + 1$

10. Use Representations to Communicate Mathematical Ideas (1)(E) Compare the graphs of the functions $f(x) = 2(x-1)^2 + 5$ and $g(x) = -2(x-1)^2 + 5$. In your answer, use the concepts of vertex, range, minimum, and maximum.

Rewrite these functions from vertex form to standard form. Then use either form to calculate $f(1)$ and $f(-1)$.

11. $f(x) = 2(x+7)^2 - 2$

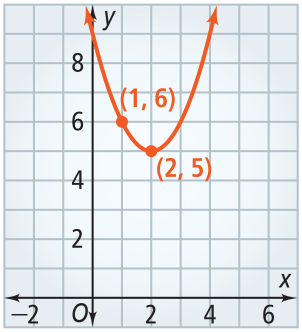
12. $f(x) = -\frac{1}{2}(x + \frac{3}{2})^2 + \frac{9}{8}$

13. $f(x) = 3(x-3)^2 + 2.5$

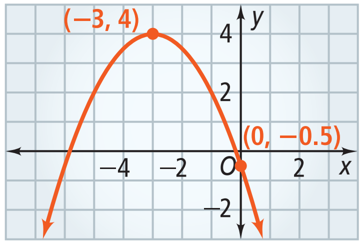
14. Explain Mathematical Ideas (1)(G) Does the graph of a quadratic function depend on whether it is written in vertex form or standard form? Explain.

For each graph, write a quadratic function in vertex form and standard form.

15.



16.



For each description, write a quadratic function in vertex form and standard form.

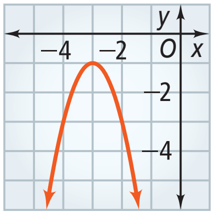
17. The parabola with vertex (3, −2) and that passes through (4, −2.5)

18. The parabola with vertex (0, 5) and that passes through the point (−3, −13)

19. The parabola with vertex (−10, 0) and that passes through (1, 60.5)

TEXAS End-of-Course PRACTICE

20. Which equation describes the parabola shown in the graph?



A. $f(x) = -2(x-3)^2 - 1$

B. $f(x) = (x-3)^2 - 1$

C. $f(x) = -(x+3)^2$

D. $f(x) = -2(x+3)^2 - 1$

21. Which function is equivalent to $f(x) = -(x-4)^2 + 4$?

F. $f(x) = -x^2 + 8x$

G. $f(x) = -x^2 + 8x - 12$

H. $f(x) = -x^2 + 20$

J. $f(x) = x^2 - 8x + 8$

22. Find the range of the function $f(x) = -3(x+2.5)^2 + 5.9$.

A. $f(x) \leq 5.9$

B. $f(x) \geq 5.9$

C. $f(x) \leq -2.5$

D. $f(x) \geq 3.4$

23. Identify the vertex and one other point on the graph of the function $f(x) = -(x+1)^2 - 4$. Then use the axis of symmetry to identify a second point on the graph.

8-5 Solving Quadratic Equations

TEKS FOCUS

TEKS (8)(A) Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(D), (7)(A)

VOCABULARY

- **Quadratic equation** – A quadratic equation is one that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.
- **Root of an equation** – a solution of an equation of the form $f(x) = 0$
- **Standard form of a quadratic equation** – The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$.
- **Zero of a function** – A zero of a function $f(x)$ is any value of x for which $f(x) = 0$.
- **Number sense** – the understanding of what numbers mean and how they are related

ESSENTIAL UNDERSTANDING

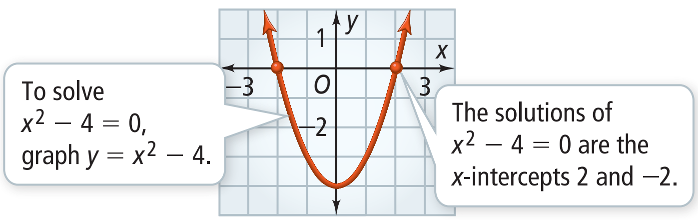
Quadratic equations can be solved by a variety of methods, including graphing and finding square roots.

take note Key Concept Standard Form of a Quadratic Equation

A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. This form is called the **standard form of a quadratic equation**.

take note Key Concept Solve Quadratic Equations by Graphing

One way to solve a quadratic equation $ax^2 + bx + c = 0$ is to graph the related quadratic function $y = ax^2 + bx + c$. The solutions of the equation are the x-intercepts of the related function.



A quadratic equation can have two, one, or no real-number solutions. In a future course you will learn about solutions of quadratic equations that are not real numbers. In this course, *solutions* refers to real-number solutions.

The solutions of a quadratic equation and the x-intercepts of the graph of the related function are often called **roots of the equation** or **zeros of the function**.

Problem 1

TEKS Process Standard (1)(D)

Solving by Graphing

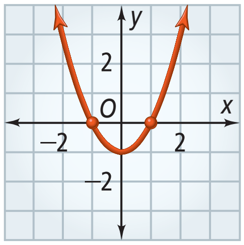
What are the solutions of each equation? Use a graph of the related function.

Think

What feature of the graph shows the solutions of the equation?
The x-intercepts show the solutions of the equation.

A $x^2 - 1 = 0$

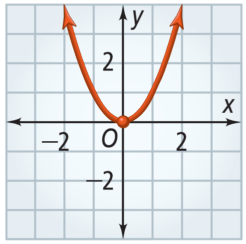
Graph $y = x^2 - 1$.



There are two solutions, ± 1.

B $x^2 = 0$

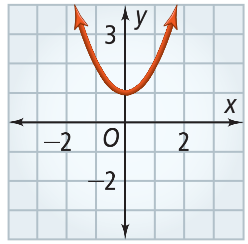
Graph $y = x^2$.



There is one solution, 0.

C $x^2 + 1 = 0$

Graph $y = x^2 + 1$.



There is no real-number solution.

Problem 2

TEKS Process Standard (1)(C)

Using the Graph of a Quadratic Function

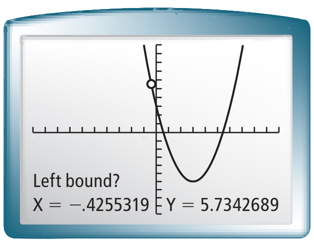
A **What tool would you use to solve the quadratic equation $x^2 - 6x + 3 = 0$? Explain your choice.**

The solutions of the equation are the zeros of the function $(f)(x) = x^2 - 6x + 3$. A graphing calculator is a good tool to use to identify the zeros of a quadratic function, which correspond to the x-intercepts of the graph of the function. If you graph the function with paper and pencil, you may not be able to determine the intercepts very accurately. When you use technology, you can get a good approximation.

B **Use a graphing calculator to solve $x^2 - 6x + 3 = 0$.**

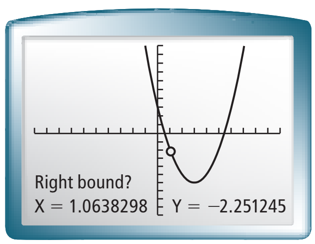
Step 1 Enter $y = x^2 - 6x + 3$ on the **Y =** screen. Use the **CALC** feature. Select **ZERO**.

Step 2



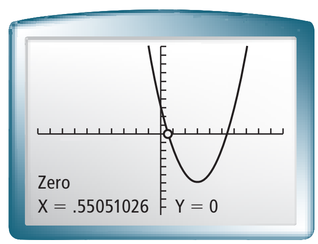
Move the cursor to the left of the first x-intercept. Press enter to set the left bound.

Step 3



Move the cursor slightly to the right of the intercept. Press enter to set the right bound.

Step 4



Press enter to display the first zero, which is about 0.55.

Repeat Steps 2–4 to identify the second zero from the other x-intercept. The second zero is about 5.45. So the solutions of the equation are about 0.55 and 5.45.

Problem 3

Solving Using Square Roots

What are the solutions of $3x^2 - 75 = 0$?

Plan

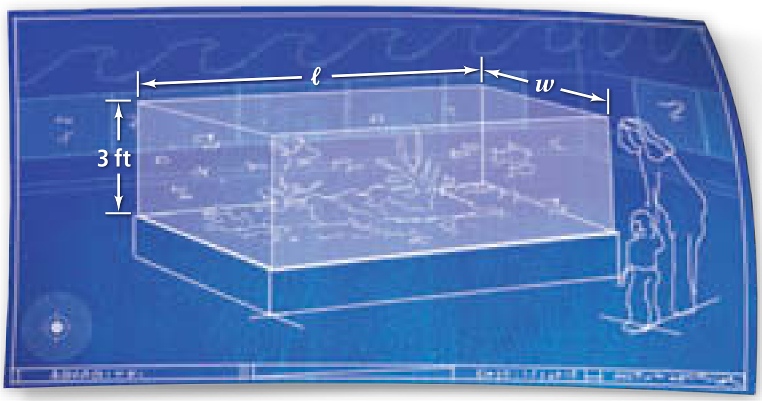
How do you know you can solve using square roots?
The equation has an x^2-term and a constant term, but no x-term. So, you can write the equation in the form $x^2 = k$ and then find the square roots of each side.

Think	Write
Write the original equation.	$3x^2 - 75 = 0$
Isolate x^2 on one side of the equation.	$3x^2 = 75$ $x^2 = 25$
Find the square roots of each side and simplify.	$x = \pm\sqrt{25}$ $x = \pm 5$

Problem 4

Choosing a Reasonable Solution

Aquarium **An aquarium is designing a new exhibit to showcase tropical fish. The exhibit will include a tank that is a rectangular prism with a length ℓ that is twice the width w. The volume of the tank is 420 ft^3. What is the width of the tank to the nearest tenth of a foot?**



Plan

How can you write the length of the tank?
The length ℓ is twice the width w, so write the length as $2w$.

$V = \ell w h$	Use the formula for volume of a rectangular prism.
$420 = (2w)(w)(3)$	Substitute 420 for V, $2w$ for ℓ, and 3 for h.
$420 = 6w^2$	Simplify.
$70 = w^2$	Divide each side by 6.
$\pm\sqrt{70} = w$	Find the square roots of each side.
$\pm 8.366600265 \approx w$	Use a calculator.

A tank cannot have a negative width, so only the positive square root makes sense. The tank will have a width of about 8.4 ft.

PRACTICE and APPLICATION EXERCISES

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Use a graphing calculator to graph the function and approximate the x-intercepts. Then write the zeros of the function. Round to the nearest hundredth.

1. $f(x) = x^2 - 6x - 16$
2. $f(x) = x^2 - 18x + 5$
3. $f(x) = 2x^2 + x - 6$

4. **Select Tools to Solve Problems (1)(C)** Explain why a graphing calculator is a good tool to use to find the x-intercepts of the function $f(x) = 0.25x^2 - 8x - 45$. Use a graphing calculator to graph the function and approximate the x-intercepts to the nearest hundredth. What are the zeros of the function?

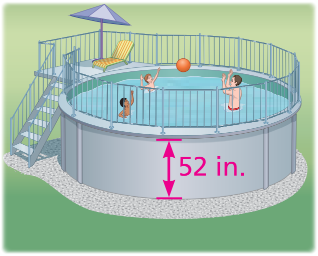
5. **Apply Mathematics (1)(A)** You have enough paint to cover an area of 50 ft^2. What is the side length of the largest square that you could paint? Round your answer to the nearest tenth of a foot.

6. **Apply Mathematics (1)(A)** You have enough shrubs to cover an area of 100 ft^2. What is the radius of the largest circular region you can plant with these shrubs? Round your answer to the nearest tenth of a foot.

Tell how many solutions each equation has.

7. $h^2 = -49$
8. $c^2 - 18 = 9$
9. $s^2 - 35 = -35$

10. **Apply Mathematics (1)(A)** A circular aboveground pool has a height of 52 in. and a volume of 1100 ft^3. What is the radius of the pool to the nearest tenth of a foot? Use the equation $V = \pi r^2 h$, where V is the volume, r is the radius, and h is the height.



11. **Explain Mathematical Ideas (1)(G)** For what values of n will the equation $x^2 = n$ have two solutions? Exactly one solution? No solution?

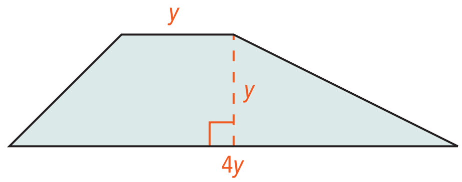
Use Multiple Representations to Communicate Mathematical Ideas (1)(D) **Solve each equation by graphing the related function. If the equation has no real-number solution, write *no solution*.**

12. $x^2 - 9 = 0$
13. $x^2 + 7 = 0$
14. $3x^2 = 0$
15. $3x^2 - 12 = 0$
16. $x^2 + 4 = 0$
17. $\frac{1}{3}x^2 - 3 = 0$

Model each problem with a quadratic equation. Then solve. If necessary, round to the nearest tenth.

18. Find the length of a side of a square with an area of 169 m^2.
19. Find the length of a side of a square with an area of 75 ft^2.
20. Find the radius of a circle with an area of 90 cm^2.

21. The trapezoid has an area of 1960 cm^2. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$, where A represents the area of the trapezoid, h represents its height, and b_1 and b_2 represent its bases, to find the value of y.



Solve each equation by finding square roots. If the equation has no real-number solution, write *no solution*.

22. $n^2 = 81$

23. $a^2 = 324$

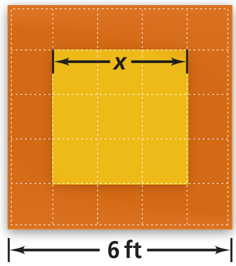
24. $k^2 - 196 = 0$

25. $r^2 + 49 = 49$

26. $w^2 - 36 = -64$

27. $4g^2 = 25$

28. **Apply Mathematics (1)(A)** You are making a square quilt with the design shown at the right. Find the side length x of the inner square that would make its area equal to 50% of the total area of the quilt. Round to the nearest tenth of a foot.



Solve each equation by finding square roots. If the equation has no real-number solution, write *no solution*. If a solution is irrational, round to the nearest tenth.

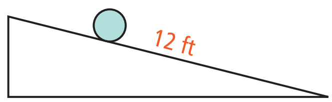
29. $1.2z^2 - 7 = -34$

30. $49p^2 - 16 = -7$

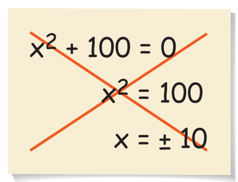
31. $3m^2 - \frac{1}{12} = 0$

32. Find the value of c such that the equation $x^2 - c = 0$ has 12 and -12 as solutions.

STEM 33. **Apply Mathematics (1)(A)** The equation $d = \frac{1}{2}at^2$ gives the distance d that an object starting at rest travels given acceleration a and time t. Suppose a ball rolls down the ramp shown at the right with acceleration $a = 2 \text{ ft/s}^2$. Find the time it will take the ball to roll from the top of the ramp to the bottom. Round to the nearest tenth of a second.



34. **Analyze Mathematical Relationships (1)(F)** Describe and correct the error made in solving the equation.



35. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** Write and solve an equation in the form $ax^2 + c = 0$, where $a \neq 0$, that satisfies the given condition.

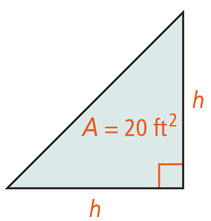
a. The equation has no solution.

b. The equation has exactly one solution.

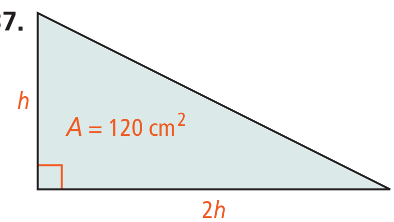
c. The equation has two solutions.

Find the value of *h* for each triangle. If necessary, round to the nearest tenth.

36.



37.



38. **a.** Solve the equation $(x-7)^2 = 0$.

b. Find the vertex of the graph of the related function $y = (x-7)^2$.

c. Choose a value for h and repeat parts (a) and (b) using $(x-h)^2 = 0$ and $y = (x-h)^2$.

d. Where would you expect to find the vertex of the graph of $y = (x+4)^2$? Explain.

39. **Select Tools to Solve Problems (1)(C)** You can use a spreadsheet like the one at the right to solve a quadratic equation.

a. What spreadsheet formula would you use to find the value in cell B2?

b. Use a spreadsheet to find the solutions of the quadratic equation $6x^2 - 24 = 0$. Explain how you used the spreadsheet to find the solutions.

c. Suppose a quadratic equation has solutions that are not integers. How could you use a spreadsheet to approximate the solutions?

	A	B
1	x	6x^2 − 24 = 0
2	−3	■
3	−2	■
4	−1	■
5	0	■
6	1	■
7	2	■
8	3	■

TEXAS End-of-Course PRACTICE

40. A package is shaped like a rectangular prism. The length and the width are equal. The volume of the package is 32 ft^3. The height is 2 ft. What is its length?

A. −4 ft **B.** 4 ft **C.** 8 ft **D.** 16 ft

41. What is the y-intercept of the line with equation $y = 3x - 4$?

F. −4 **G.** −3 **H.** 3 **J.** 4

42. What is the domain of the relation $\{(3, -1), (4, 2), (-2, 5), (1, 0)\}$?

A. $\{-1, 0, 2, 5\}$ **B.** $\{0, 2, 5\}$ **C.** $\{-2, 1, 3, 4\}$ **D.** $\{1, 3, 4\}$

43. What is the solution of the inequality $-3x + 2 \leq 14$?

F. $x \leq -4$ **G.** $x \geq -4$ **H.** $x \leq 4$ **J.** $x \geq 4$

44. The surface area of a cube is 96 ft^2.

a. What is the length of each edge? Show your work.

b. Suppose you double the length of each edge. What happens to the surface area of the cube? Show your work.

8-6 Factoring to Solve Quadratic Equations

TEKS FOCUS

TEKS (8)(A) Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(A)

VOCABULARY

- **Zero-Product Property** – For all real numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$.
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

You can solve some quadratic equations $ax^2 + bx + c = 0$, including equations where $b \neq 0$, by using the *Zero-Product Property*.

take note

Property Zero-Product Property

For any real numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$.

Example If $(x + 3)(x + 2) = 0$, then $x + 3 = 0$ or $x + 2 = 0$.

Problem 1 TEKS Process Standard (1)(E)

Using the Zero-Product Property

What are the solutions of the equation $(4t + 1)(t - 2) = 0$?

$(4t + 1)(t - 2) = 0$

$4t + 1 = 0$ or $t - 2 = 0$ Use the Zero-Product Property.

$4t = -1$ or $t = 2$ Solve for t.

$t = -\frac{1}{4}$ or $t = 2$

The solutions are $-\frac{1}{4}$ and 2.

Think

How else can you write the solutions?
You can write the solutions using set notation: $\{-\frac{1}{4}, 2\}$.

Problem 2

Solving by Factoring

Multiple Choice **What are the solutions of the equation $x^2 + 8x + 15 = 0$?**

Ⓐ $-5, -3$ Ⓒ $-3, 5$

Ⓑ $-5, 3$ Ⓓ $3, 5$

Plan

How can you factor $x^2 + 8x + 15$?
Find two integers with a product of 15 and a sum of 8.

$x^2 + 8x + 15 = 0$

$(x + 3)(x + 5) = 0$ Factor $x^2 + 8x + 15$.

$x + 3 = 0$ or $x + 5 = 0$ Use the Zero-Product Property.

$x = -3$ or $x = -5$ Solve for x.

The solutions are -3 and -5. The correct answer is A.

Problem 3

Writing in Standard Form First

What are the solutions of $4x^2 - 21x = 18$?

Think

Why do you need to subtract 18 from each side before you factor?
To use the Zero-Product Property, one side of the equation must be zero.

$4x^2 - 21x = 18$

$4x^2 - 21x - 18 = 0$ Subtract 18 from each side.

$(4x + 3)(x - 6) = 0$ Factor $4x^2 - 21x - 18$.

$4x + 3 = 0$ or $x - 6 = 0$ Use the Zero-Product Property.

$4x = -3$ or $x = 6$ Solve for x.

$x = -\frac{3}{4}$ or $x = 6$

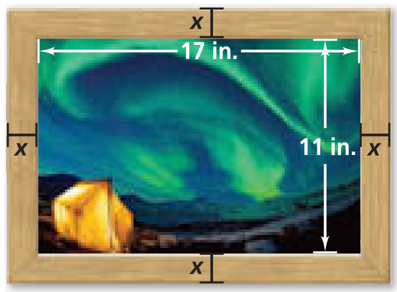
The solutions are $-\frac{3}{4}$ and 6.

Problem 4

TEKS Process Standard (1)(A)

Using Factoring to Solve a Real-World Problem

Photography **You are constructing a frame for the rectangular photo shown. You want the frame to be the same width all the way around and the total area of the frame and photo to be 315 in.^{2}. What should the outer dimensions of the frame be?**



Know
The size of the photo is 11 in. by 17 in. The total area is 315 in.^{2}.

Need
The outer dimensions of the frame

Plan
Write the frame's outer dimensions in terms of its width x. Use these dimensions to write an equation for the area of the frame and photo.

continued on next page ▶

Problem 4 *continued*

$(2x + 11)(2x + 17) = 315$	Width × Length = Area
$4x^2 + 56x + 187 = 315$	Find the product $(2x + 11)(2x + 17)$.
$4x^2 + 56x - 128 = 0$	Subtract 315 from each side.
$4(x^2 + 14x - 32) = 0$	Factor out 4.
$4(x + 16)(x - 2) = 0$	Factor $x^2 + 14x - 32$.
$x + 16 = 0$ or $x - 2 = 0$	Use the Zero-Product Property.
$x = -16$ or $x = 2$	Solve for x.

The only reasonable solution is 2. So the outer dimensions of the frame are $2(2) + 11$ in. by $2(2) + 17$ in., or 15 in. by 21 in.

Think

Why can you ignore the factor of 4?
By the Zero-Product Property, one of the factors, 4, $x + 16$, or $x - 2$, must equal 0. Since $4 \neq 0$, either $x + 16$ or $x - 2$ equals 0.

PRACTICE and APPLICATION EXERCISES

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Use Representations to Communicate Mathematical Ideas (1)(E) **Use the Zero-Product Property to solve each equation.**

1. $(x - 9)(x - 8) = 0$ **2.** $(4k + 5)(k + 7) = 0$ **3.** $n(n + 2) = 0$

4. Explain Mathematical Ideas (1)(G) Describe and correct the error made in solving the equation.

$2x^2 + 3x = 20$
$x(2x + 3) = 20$
$x = 0$ or $2x + 3 = 0$
$x = 0$ or $x = -\frac{3}{2}$

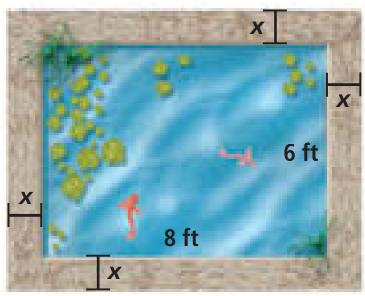
5. A box shaped like a rectangular prism has a volume of 280 in.^{3}. Its dimensions are 4 in. by $(n + 2)$ in. by $(n + 5)$ in. Find n.

6. Explain Mathematical Ideas (1)(G) How many solutions does an equation of the form $x^2 - k^2 = 0$ have? Explain.

7. Apply Mathematics (1)(A) You are knitting a blanket. You want the area of the blanket to be 24 ft^2. You want the length of the blanket to be 2 ft longer than its width. What should the dimensions of the blanket be?

STEM **8. Apply Mathematics (1)(A)** You are building a rectangular deck. The area of the deck should be 250 ft^2. You want the length of the deck to be 5 ft longer than twice its width. What should the dimensions of the deck be?

9. Apply Mathematics (1)(A) You have a rectangular koi pond that measures 6 ft by 8 ft. You have enough concrete to cover 72 ft^2 for a walkway, as shown in the diagram. What should the width of the walkway be?



10. Find the zeros of the function $f(x) = x^2 - 3x + 2$ by factoring. How can you verify the zeros of the function are correct by looking at the graph?

Write each equation in standard form. Then solve.

11. $7n^2 + 16n + 15 = 2n^2 + 3$

12. $4q^2 + 3q = 3q^2 - 4q + 18$

Solve by factoring.

13. $x^2 + 11x + 10 = 0$

14. $2z^2 - 21z - 36 = 0$

15. $p^2 - 4p = 21$

16. $c^2 = 5c$

17. $2w^2 - 11w = -12$

18. $9b^2 = 16$

19. **Use a Problem-Solving Model (1)(B)** You throw a softball into the air with an initial upward velocity of 38 ft/s and an initial height of 5 ft.

a. Use the vertical motion model to write an equation that gives the ball's height h, in feet, at time t, in seconds.

b. The ball's height is 0 ft when it is on the ground. Solve the equation you wrote in part (a) for $h = 0$ to find when the ball lands.

Solve each cubic equation by factoring out the GCF first.

20. $x^3 - 10x^2 + 24x = 0$

21. $x^3 - 5x^2 + 4x = 0$

Solve. Factor by grouping.

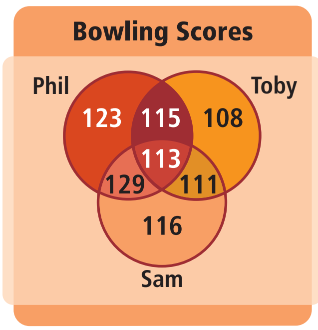
22. $x^3 + 5x^2 - x - 5 = 0$

23. $x^3 + x^2 - 4x - 4 = 0$

24. $x^3 + 2x^2 - 9x - 18 = 0$

TEXAS End-of-Course PRACTICE

25. Phil, Toby, and Sam bowled four games last weekend. Their scores are shown in the Venn diagram at the right. What is the highest score that only Toby and Sam have in common?



26. What is the negative solution of the equation $2x^2 - 13x - 7 = 0$?

27. What is the y-intercept of the line with equation $3y - 4x = 9$?

28. A rectangular card has a length 1 in. longer than twice the width and an area of 15 in.^{2}. What is the width of the card, in inches?

8-7 Writing Quadratic Functions

TEKS FOCUS

TEKS (6)(C) Write quadratic functions when given real solutions and graphs of their related equations.

TEKS (1)(B) Use a problem-solving model that incorporates analyzing given information, **formulating** a plan or **strategy**, determining a solution, justifying the solution, and evaluating the problem-solving process and the **reasonableness** of the solution.

Additional TEKS (1)(D), (7)(B)

VOCABULARY

- **Linear factor** – A linear factor of an expression is a factor in the form $ax + b$, $a \neq 0$.

- **Formulate** – create with careful effort and purpose. You can formulate a plan or strategy to solve a problem.
- **Reasonableness** – the quality of being within the realm of common sense or sound reasoning. The reasonableness of a solution is whether or not the solution makes sense.
- **Strategy** – a plan or method for solving a problem.

ESSENTIAL UNDERSTANDING

You can use the solutions of a quadratic equation to write an equation for a related quadratic function.

take note — Key Concept Relating Linear Factors, Solutions, and Zeros

Each linear factor of a quadratic expression corresponds to a solution of a related quadratic equation and a zero of a related quadratic function.

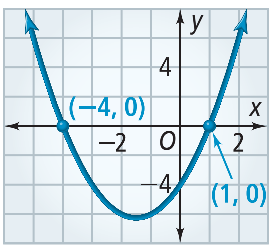
quadratic expression	quadratic equation	quadratic function
$x^2 + 3x - 4$ $(x + 4)(x - 1)$	$x^2 + 3x - 4 = 0$	$f(x) = x^2 + 3x - 4$

You use the linear factors of the quadratic expression to find the solutions of the quadratic equation.

$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -4 \quad \text{or} \quad x = 1$$

The solutions of the equation are the zeros of the related quadratic function, and the x-intercepts of its graph.

$$f(x) = x^2 + 3x - 4$$
$$f(-4) = (-4)^2 + 3(-4) - 4 = 16 - 12 - 4 = 0$$
$$f(1) = (1)^2 + 3(1) - 4 = 1 + 3 - 4 = 0$$



Key Concept Writing Quadratic Functions

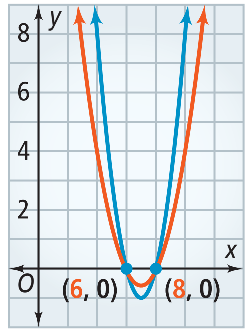
take note

If a real number r is a solution of a quadratic equation in standard form, then $x - r$ is a factor of a related quadratic function.

Since each solution r corresponds to a zero of the related quadratic function, you can use the zeros to find the factors of the function.

For example, suppose a quadratic function $y = f(x)$ has zeros 6 and 8. Then $x - 6$ and $x - 8$ are factors of $f(x)$. So the factored form of any quadratic function with zeros 6 and 8 can be written as shown below.

$$f(x) = a(x - 6)(x - 8)$$



Problem 1

Relating Linear Factors and Zeros

A Describe the relationship between the linear factors of the quadratic expression $x^2 - 2x - 15$ and the zeros of the related quadratic function $f(x) = x^2 - 2x - 15$.

The zeros of the quadratic function $f(x) = x^2 - 2x - 15$ are the solutions of the quadratic equation $0 = x^2 - 2x - 15$. You can solve $0 = x^2 - 2x - 15$ by factoring the expression $x^2 - 2x - 15$ and using the Zero-Product Property, so each linear factor of $x^2 - 2x - 15$ leads to one zero of the function.

$$0 = x^2 - 2x - 15$$

$$0 = (x - 5)(x + 3)$$

$$x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 5 \quad \text{or} \quad x = -3$$

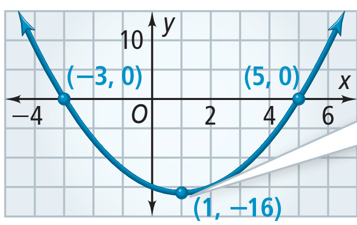
The zeros of the function $f(x) = x^2 - 2x - 15$ are 5 and -3.

Think

In addition to x-intercepts and the vertex, what other point helps you sketch the graph?
When $x = 0$, $y = -15$. The y-intercept is $(0, -15)$.

B Use the solutions of $0 = x^2 - 2x - 15$ that you found in part (A) to help you graph $y = x^2 - 2x - 15$.

The solutions of the equation, 5 and -3, are the zeros of the function $y = x^2 - 2x - 15$ and the x-intercepts of its graph. Plot the x-intercepts and the vertex. Then sketch the parabola.



The x-coordinate of the vertex is $-\frac{b}{2a} = 1$, and the y-coordinate is $1^2 - 2(1) - 15 = -16$.

Problem 2 TEKS Process Standard (1)(B)

Using Solutions to Write Quadratic Functions

The solutions of a quadratic equation are −2 and 8. What is the standard form of a related quadratic function?

Analyze Given Information You know the solutions of the equation and need to use this to write the function in standard form.

Formulate a Plan Use each solution r to write one linear factor $x - r$ of the function. Then multiply the two linear factors to find the standard form of the quadratic function.

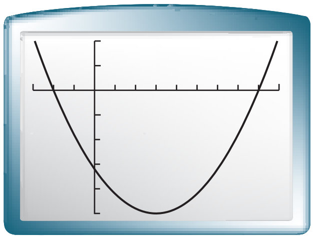
Determine a Solution The solutions are −2 and 8. Each solution r corresponds to a linear factor $(x - r)$.

Think

Is this the only quadratic function with zeros of −2 and 8?
No; every function of the form $f(x) = a(x + 2)(x - 8)$, where $a \neq 0$ has zeros of −2 and 8.

$f(x) = (x - (-2))(x - 8)$	Write the function in factored form.
$f(x) = (x + 2)(x - 8)$	Simplify.
$f(x) = x^2 + 2x - 8x - 16$	Multiply the factors.
$f(x) = x^2 - 6x - 16$	Simplify to write the function in standard form.

Justify the Solution Check your work by graphing $f(x) = x^2 - 6x - 16$ and verifying that the x-intercepts of the graph match the given solutions of the equation.



x scale: 1 y scale: 5

The x-intercepts of the parabola are −2 and 8, which are the given solutions of the equation.

Evaluate the Problem–Solving Process The plan to use the solutions to write a function in factored form and then rewriting the function in standard form worked. Checking the solution by graphing confirms that the function is correct.

You can also check by substituting −2 and 8 in the standard form of the equation to see if both values satisfy the equation.

$(-2)^2 - 6(-2) - 16 = 0$	$(8)^2 - 6(8) - 16 = 0$
$4 + 12 - 16 = 0$	$64 - 48 - 16 = 0$
$0 = 0$ ✔	$0 = 0$ ✔

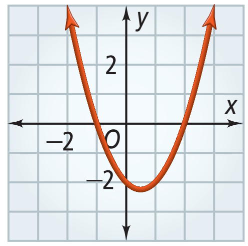
Both values make the equation true.

Problem 3

TEKS Process Standard (1)(B)

Using a Graph to Write a Quadratic Function

What is a quadratic function in standard form for the parabola shown?



Analyze Given Information The graph gives the x-intercepts, which are the zeros of the function, and shows that $f(0) = -2$.

Formulate a Plan Use the zeros to write the factored form of the function.

Determine a Solution The zeros are −1 and 2.

$f(x) = a(x - (-1))(x - 2)$ Write the function in factored form.

$f(x) = a(x^2 - x - 2)$ Mulitply and simplify.

Use the fact that $f(0) = -2$ to solve for a.

$f(0) = a(0^2 - 0 - 2)$ Substitute 0 for x.

$-2 = -2a$ Substitute −2 for $f(0)$ and simplify.

$1 = a$ Solve for a.

So $f(x) = x^2 - x - 2$.

Justify the Solution Check the solution by graphing the function in standard form on a graphing calculator.

Evaluate the Reasonableness of the Solution The graph of the function in standard form is the same as the graph above. The solution is reasonable.

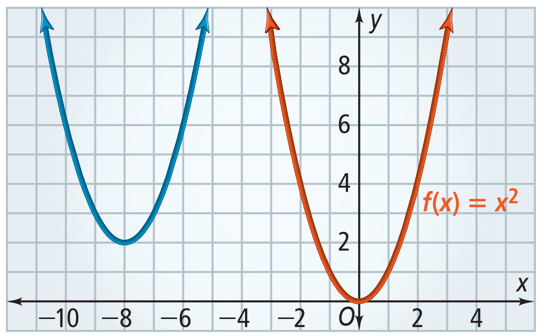
Think

Why do you multiply the product of the factors by a constant a?
There are infinitely many quadratic functions with roots −1 and 2, each determined by a unique constant factor.

Problem 4

Writing a Quadratic Function Using Transformations

The blue parabola is a translation of the graph of the parent quadratic function $f(x) = x^2$. What is the function written in standard form for the translated parabola?



A translation shifts the parent function $f(x) = x^2$ to the left 8 units and up 2 units.

$y = (x + 8)^2 + 2$ Write the equation in vertex form.

$y = x^2 + 16x + 66$ Simplify.

A quadratic function in standard form for the blue parabola is $h(x) = x^2 + 16x + 66$.

Think

What is the vertex form of a quadratic function?
If the vertex of the function $y = x^2$ is shifted to (h, k), then the equation of the transformed parabola is $y = (x - h)^2 + k$.

PRACTICE and APPLICATION EXERCISES

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1. **Connect Mathematical Ideas (1)(F)** What are the linear factors of the quadratic expression $x^2 + 14x + 40$? What are the zeros of the related quadratic function? Explain the relationship between the factors of the expression and the zeros of the function.

2. **Create Representations to Communicate Mathematical Ideas (1)(E)** What are the solutions of $x^2 + 14x - 15 = 0$? Use the solutions to help you graph the related quadratic function.

Tell whether each of the following is a *quadratic expression*, a *quadratic equation*, or a *quadratic function*. State the linear factors of any expressions, the solutions of any equations, and the zeros of any functions.

3. $f(x) = x^2 + 14x + 40$

4. $x^2 - 14x - 120$

5. $x^2 + 8x + 16 = 0$

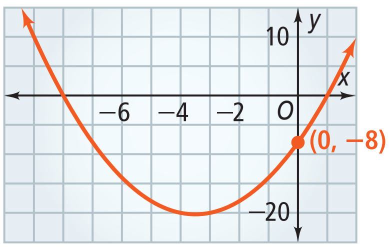
6. $0 = x^2 - 5x + 4$

7. $x^2 - 7x - 60$

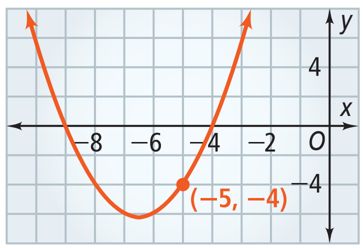
8. $y = x^2 + 9x - 22$

Use the x-intercepts of the parabola and the given point to write a quadratic function in factored form.

9.



10.



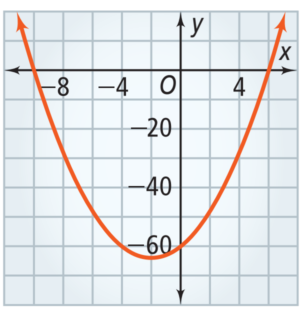
The solutions of a quadratic equation are given. Write a related quadratic function in factored form.

11. 3, 7

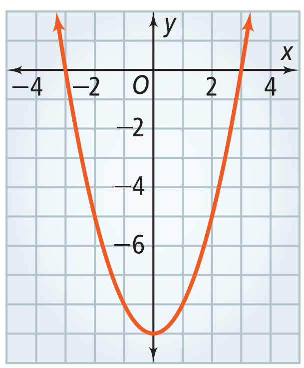
12. −2, 5

Write a quadratic function in standard form for the parabola shown.

13.



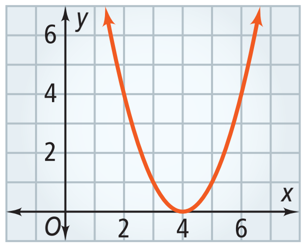
14.



15. **Use a Problem-Solving Model (1)(B)** The solutions of a quadratic equation are $-\sqrt{7}$ and $\sqrt{7}$. Write a related quadratic function in standard form. Use a problem-solving model by

- analyzing the given information,
- formulating a plan or strategy,
- determining a solution,
- justifying the solution, and
- evaluating the problem-solving process.

16. **Evaluate Reasonableness (1)(B)** Write a quadratic function in standard form for the parabola at the right. Use a problem-solving model by



- analyzing the given information,
- formulating a plan or strategy,
- determining a solution,
- justifying the solution, and
- evaluating the reasonableness of the solution.

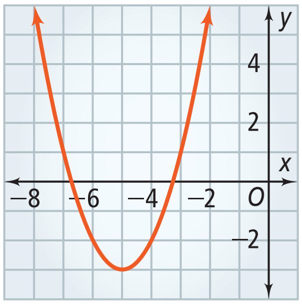
17. **Select Tools to Solve Problems (1)(C)** Describe two different methods you could use to sketch a graph of a quadratic function with zeros -8 and 3, and y-intercept -2. Which tool(s) would you need for each method?

18. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** Explain the relationship among the terms *solution, zero,* and *linear factor* as they relate to quadratic equations, functions, and expressions.

19. **Apply Mathematics (1)(A)** The height h, in feet, of a coin being tossed at a football game is given by the function $h = -16t^2 + 12t + 4$, where t is the time in seconds. Sketch the graph of this function. When will the coin hit the ground?

Use translations of the parent function $f(x) = x^2$ to write a quadratic function in standard form for each parabola below.

20.



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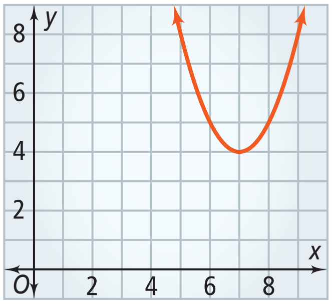


TEXAS End-of-Course PRACTICE

22. What are the solutions of the equation $x^2 - 13x + 40 = 0$?

A. -10 and -3

B. 8 and 5

C. 8 and -5

D. 10 and -4

23. Which function has a graph with x-intercepts 1 and -5?

F. $f(x) = x^2 + 4x - 5$

G. $f(x) = x^2 - 4x - 5$

H. $f(x) = x^2 + 6x + 5$

J. $f(x) = x^2 - 6x + 5$

24. Which function can be graphed by shifting the graph of $y = x^2$ right 3 units and down 4 units?

A. $f(x) = (x + 3)^2 + 4$

B. $f(x) = (x - 4)^2 + 3$

C. $f(x) = (x + 4)^2 - 3$

D. $f(x) = (x - 3)^2 - 4$

25. Sketch the graph of the function $f(x) = x^2 + 9x + 18$. Use the graph to find the zeros of the function.

26. Explain how to use the solutions of a quadratic equation to write an equation of a related quadratic function in standard form. Is the function unique? Explain.

8-8 Completing the Square

TEKS FOCUS

TEKS (8)(A) Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

TEKS (1)(E) Create and use **representations** to organize, record, and communicate mathematical ideas.

Additional TEKS (1)(A)

VOCABULARY

- **Completing the square** – a method of solving quadratic equations. Completing the square is a way to rewrite a quadratic equation into the form $m^2 = n$.
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

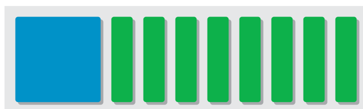
ESSENTIAL UNDERSTANDING

You can solve any quadratic equation by first writing it in the form $m^2 = n$.

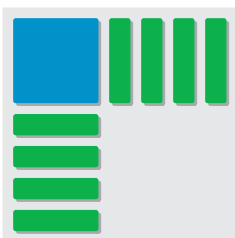
take note

Key Concept Completing the Square

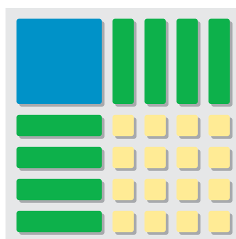
You can model a quadratic expression using algebra tiles. The algebra tiles below represent the expression $x^2 + 8x$.



Here is the same expression rearranged to form part of a square. Notice that the x-tiles have been split evenly into two groups of four.



You can complete the square by adding 4^2, or 16, 1-tiles. The completed square is $x^2 + 8x + 16$, or $(x + 4)^2$.



In general, you can change the expression $x^2 + bx$ into a perfect-square trinomial by adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$. This process is called **completing the square.** The process is the same whether b is positive or negative.

Think

Can c be negative?
No. c is the square of a real number, which is never negative.

Problem 1

Finding c to Complete the Square

What is the value of c such that $x^2 - 16x + c$ is a perfect-square trinomial?

The value of b is -16.

The term to add to $x^2 - 16x$ is $\left(\frac{-16}{2}\right)^2$, or 64. So $c = 64$.

Problem 2

Solving $x^2 + bx + c = 0$

What are the solutions of the equation $x^2 - 14x + 16 = 0$?

$x^2 - 14x + 16 = 0$	
$x^2 - 14x = -16$	Subtract 16 from each side.
$x^2 - 14x + 49 = -16 + 49$	Add $\left(\frac{-14}{2}\right)^2$, or 49, to each side.
$(x - 7)^2 = 33$	Write $x^2 - 14x + 49$ as a square.
$x - 7 = \pm\sqrt{33}$	Find square roots of each side.
$x - 7 \approx \pm 5.74$	Use a calculator to approximate $\sqrt{33}$.
$x - 7 \approx 5.74$ or $x - 7 \approx -5.74$	Write as two equations.
$x \approx 5.74 + 7$ or $x \approx -5.74 + 7$	Add 7 to each side.
$x \approx 12.74$ or $x \approx 1.26$	Simplify.

Think

Why do you write $x - 7 \approx \pm 5.74$ as two equations?
Recall that the symbol $\pm$ means "plus or minus." That means $x - 7$ equals 5.74 or -5.74.

Problem 3

TEKS Process Standard (1)(E)

Finding the Vertex by Completing the Square

Find the vertex of $y = x^2 + 6x + 8$ by completing the square.

$y = x^2 + 6x + 8$	
$y - 8 = x^2 + 6x$	Subtract 8 from each side.
$y - 8 + 9 = x^2 + 6x + 9$	Add $\left(\frac{6}{2}\right)^2$, or 9, to each side.
$y + 1 = (x + 3)^2$	Simplify the left side and write the right side as a square.
$y = (x + 3)^2 - 1$	Subtract 1 from each side so that the equation is in vertex form.

The vertex of $y = x^2 + 6x + 8$ is $(-3, -1)$.

Think

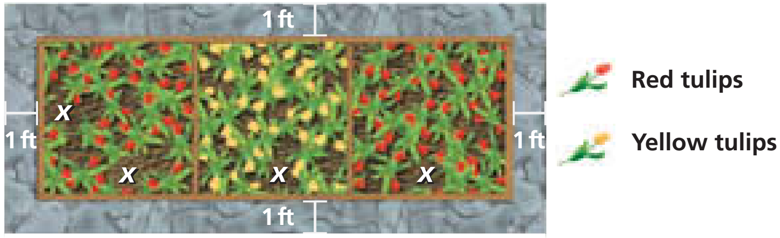
Why do you need to add the same number to each side?
You need to add the same number to each side to keep the equation balanced.

Problem 4

TEKS Process Standard (1)(A)

Completing the Square When $a \neq 1$

Gardening **You are planning a flower garden consisting of three square plots surrounded by a 1-ft border. The total area of the garden and the border is 100 ft^2. What is the side length x of each square plot?**



Know
- Area of garden and border
- Expressions for the dimensions of the garden and border

Need
The side length x of each square plot

Plan
Write and solve an equation that relates the dimensions and area of the garden and border.

Step 1 Write an equation that you can use to solve the problem.

$(3x + 2)(x + 2) = 100$	Length × Width = Area
$3x^2 + 8x + 4 = 100$	Find the product $(3x + 2)(x + 2)$.
$3x^2 + 8x = 96$	Subtract 4 from each side.
$x^2 + \frac{8}{3}x = 32$	Divide each side by 3.

Think

Why do you need to find $\frac{1}{2}\left(\frac{8}{3}\right)$?
To make a perfect-square trinomial on the left side of $x^2 + \frac{8}{3}x = 32$, find $\frac{1}{2}\left(\frac{8}{3}\right)$. Then square the result and add to each side of the equation.

Step 2 Complete the square.

$x^2 + \frac{8}{3}x + \frac{16}{9} = 32 + \frac{16}{9}$	Add $\left(\frac{4}{3}\right)^2$, or $\frac{16}{9}$, to each side.
$\left(x + \frac{4}{3}\right)^2 = \frac{304}{9}$	Write left side as a square and right side as a fraction.

Step 3 Solve the equation.

$x + \frac{4}{3} = \pm\sqrt{\frac{304}{9}}$	Find square roots of each side.
$x + \frac{4}{3} \approx \pm 5.81$	Use a calculator to approximate $\sqrt{\frac{304}{9}}$.
$x + \frac{4}{3} \approx 5.81$ or $x + \frac{4}{3} \approx -5.81$	Write as two equations.
$x \approx 4.48$ or $x \approx -7.14$	Solve for x.

The negative answer does not make sense in this problem. So, the side length of each square plot is about 4.48 ft.

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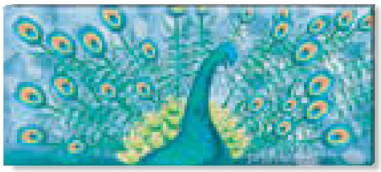
Find the value of *c* such that each expression is a perfect-square trinomial.

1. $x^2 + 18x + c$ **2.** $p^2 - 30p + c$ **3.** $k^2 - 5k + c$

Solve each equation by completing the square. If necessary, round to the nearest hundredth.

4. $g^2 + 7g = 144$ **5.** $m^2 + 16m = -59$ **6.** $w^2 - 14w + 13 = 0$

7. Apply Mathematics (1)(A) The painting shown at the right has an area of 420 in.^{2}. What is the value of x to the nearest tenth?



8. Create Representations to Communicate Mathematical Ideas (1)(E) A park is installing a rectangular reflecting pool surrounded by a concrete walkway of uniform width. The reflecting pool will measure 42 ft by 26 ft. There is enough concrete to cover 460 ft^2 for the walkway. What is the maximum width x of the walkway to the nearest tenth?

9. Explain Mathematical Ideas (1)(G) A classmate was completing the square to solve $4x^2 + 10x = 8$. For her first step she wrote $4x^2 + 10x + 25 = 8 + 25$. What was her error?

10. Explain Mathematical Ideas (1)(G) Explain why completing the square is a better strategy for solving $x^2 - 7x - 9 = 0$ than graphing or factoring.

11. Write a quadratic equation and solve it by completing the square. Show your work.

12. Apply Mathematics (1)(A) A zoo is fencing in a rectangular area for the capybaras. It plans to enclose this area using fencing on three sides, as shown at the right. The zoo has budgeted enough money for 75 ft of fencing material and would like to make an enclosure with an area of 600 ft^2.

a. Let w represent the width of the enclosure. Write an expression in terms of w for the length of the enclosure.

b. Write and solve an equation to find the width w. Round to the nearest tenth of a foot.

c. What should the length of the enclosure be?

Create Representations to Communicate Mathematical Ideas (1)(E) Find the vertex of each parabola by completing the square.

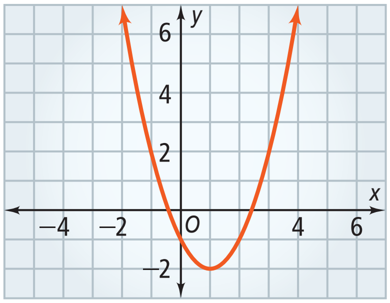
13. $y = x^2 + 4x - 16$ **14.** $y = x^2 + 6x - 7$ **15.** $y = x^2 + 2x - 28$

Solve each equation by completing the square. If necessary, round to the nearest hundredth.

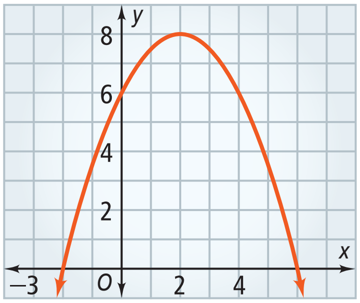
16. $4w^2 + 12w - 44 = 0$ **17.** $2y^2 - 8y - 10 = 0$ **18.** $5n^2 - 3n - 15 = 10$

Use Multiple Representations to Communicate Mathematical Ideas (1)(D) **Use each graph to estimate the values of x for which $f(x) = 5$. Then write and solve an equation to find the values of x such that $f(x) = 5$. Round to the nearest hundredth.**

19. $f(x) = x^2 - 2x - 1$



20. $f(x) = -\frac{1}{2}x^2 + 2x + 6$



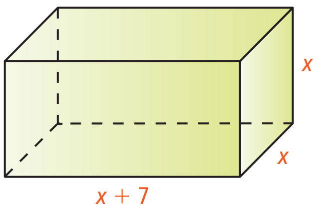
Solve each equation. If necessary, round to the nearest hundredth. If there is no real-number solution, write *no solution.*

21. $q^2 + 3q + 1 = 0$

22. $s^2 + 5s = -11$

23. $s^2 + 9s + 20 = 0$

24. Suppose the prism at the right has the same surface area as a cube with edges 8 in. long.



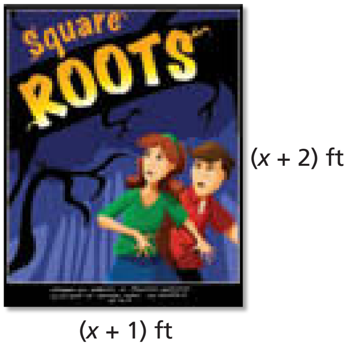
a. Write an expression for the surface area of the prism shown.

b. Write an equation that relates the surface area of the prism to the surface area of the 8-in. cube.

c. Solve the equation you wrote in part (b). What are the dimensions of the prism?

TEXAS End-of-Course PRACTICE

25. The rectangular poster has an area 40 ft^2. What is the value of x to the nearest tenth of a foot?



26. The width of a notebook is 2.15×10^{-2} m. In decimal form, how many meters wide is the notebook?

27. What is the solution of the equation $19 + x = 35$?

28. A ribbon with straight edges has an area of 24 $in.^2$. Its width is x and its length is $2x + 13$. What is the width of the ribbon in inches?

29. What is the x-intercept of the graph of $2x + 3y = 9$?

30. The sum of two numbers is 20. The difference between three times the larger number and twice the smaller number is 40. What is the larger number?

8-9 The Quadratic Formula and the Discriminant

TEKS FOCUS

TEKS (8)(A) Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(A), (1)(G)

VOCABULARY

- **Discriminant** – The discriminant of a quadratic equation of the form $ax^2 + bx + c = 0$ is $b^2 - 4ac$. The value of the discriminant determines the number of solutions of the equation.
- **Quadratic formula** – If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **Number sense** – the understanding of what numbers mean and how they are related.

ESSENTIAL UNDERSTANDING

You can find the solution(s) of *any* quadratic equation using the **quadratic formula**.

take note

Key Concept Quadratic Formula

Algebra

If $ax^2 + bx + c = 0$, and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Suppose $2x^2 + 3x - 5 = 0$. Then $a = 2$, $b = 3$, and $c = -5$. Therefore

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-5)}}{2(2)}$$

take note

Key Concept Methods for Solving a Quadratic Equation

There are many methods for solving a quadratic equation.

Method	When to Use
Graphing	Use if you have a graphing calculator handy.
Square roots	Use if the equation has no x-term.
Factoring	Use if you can factor the equation easily.
Completing the square	Use if the coefficient of x^2 is 1, but you cannot easily factor the equation.
Quadratic formula	Use if the equation cannot be factored easily or at all.

take note — Key Concept Deriving the Quadratic Formula

Here's Why It Works If you complete the square for the general equation $ax^2 + bx + c = 0$, with $a \neq 0$, you can derive the quadratic formula.

Step 1 Write $ax^2 + bx + c = 0$ so the coefficient of x^2 is 1.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{Divide each side by } a,\ a \neq 0.$$

Step 2 Complete the square.

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Subtract } \tfrac{c}{a} \text{ from each side.}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \qquad \text{Add } \left(\tfrac{b}{2a}\right)^2 \text{ to each side.}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \text{Write the left side as a square.}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \qquad \text{Multiply } -\tfrac{c}{a} \text{ by } \tfrac{4a}{4a} \text{ to get like denominators.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{Simplify the right side.}$$

Step 3 Solve the equation for x.

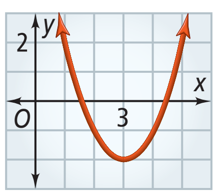
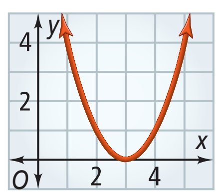
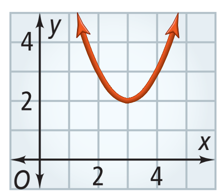
$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{Take square roots of each side.}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Simplify the right side.}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Subtract } \tfrac{b}{2a} \text{ from each side.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Simplify.}$$

take note — Key Concept Using the Discriminant of $ax^2 + bx + c = 0$

Discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Example	$x^2 - 6x + 7 = 0$ The discriminant is $(-6)^2 - 4(1)(7) = 8$, which is positive.	$x^2 - 6x + 9 = 0$ The discriminant is $(-6)^2 - 4(1)(9) = 0$.	$x^2 - 6x + 11 = 0$ The discriminant is $(-6)^2 - 4(1)(11) = -8$, which is negative.
Graph of $y = ax^2 + bx + c$			
Number of Solutions	There are two real-number solutions.	There is one real-number solution.	There are no real-number solutions.

Think

Why do you need to write the equation in standard form?
You can only use the quadratic formula with equations in the form $ax^2 + bx + c = 0$.

Problem 1

Using the Quadratic Formula

What are the solutions of $x^2 - 8 = 2x$? Use the quadratic formula.

$x^2 - 2x - 8 = 0$ — Write the equation in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Use the quadratic formula.

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$ — Substitute 1 for *a*, −2 for *b*, and −8 for *c*.

$x = \frac{2 \pm \sqrt{36}}{2}$ — Simplify.

$x = \frac{2 + 6}{2}$ or $x = \frac{2 - 6}{2}$ — Write as two equations.

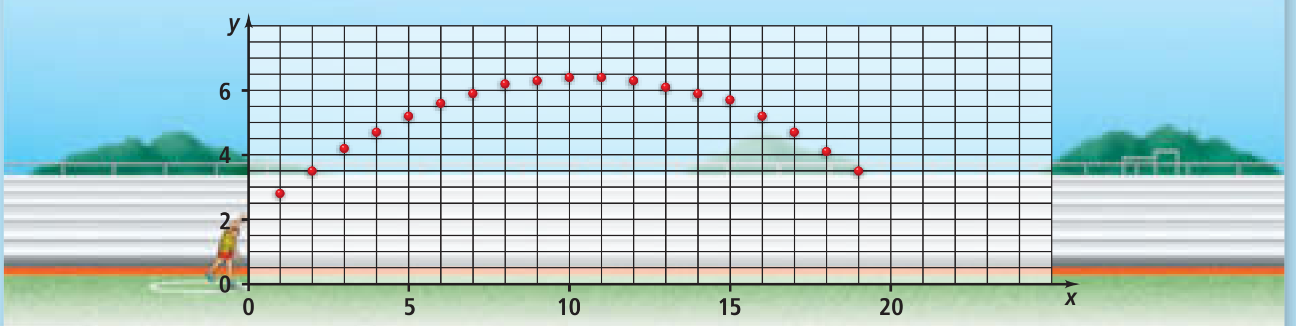
$x = 4$ or $x = -2$ — Simplify.

Problem 2

TEKS Process Standard (1)(A)

Finding Approximate Solutions

Sports **In the shot put, an athlete throws a heavy metal ball through the air. The arc of the ball can be modeled by the equation $y = -0.04x^2 + 0.84x + 2$, where x is the horizontal distance, in meters, from the athlete and y is the height, in meters, of the ball. How far from the athlete will the ball land?**



Think

Why do you substitute 0 for *y*?
When the ball hits the ground, its height will be 0.

$0 = -0.04x^2 + 0.84x + 2$ — Substitute 0 for *y* in the given equation.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Use the quadratic formula.

$x = \frac{-0.84 \pm \sqrt{0.84^2 - 4(-0.04)(2)}}{2(-0.04)}$ — Substitute −0.04 for *a*, 0.84 for *b*, and 2 for *c*.

$x = \frac{-0.84 \pm \sqrt{1.0256}}{-0.08}$ — Simplify.

$x = \frac{-0.84 + \sqrt{1.0256}}{-0.08}$ or $x = \frac{-0.84 - \sqrt{1.0256}}{-0.08}$ — Write as two equations.

$x \approx -2.16$ or $x \approx 23.16$ — Simplify.

Only the positive answer makes sense in this situation. The ball will land about 23.16 m from the athlete.

Problem 3

TEKS Process Standard (1)(C)

Choosing an Appropriate Method

Which method(s) would you choose to solve each equation? Explain your reasoning.

Using number sense and mental math to identify relationships among the coefficients of a quadratic equation can help you choose a solution method.

Think

Can you use the quadratic formula to solve part (A)?
Yes. You can use the quadratic formula with $a = 3$, $b = 0$, and $c = -9$. However, it is faster to use square roots.

A $3x^2 - 9 = 0$ — Square roots; there is no x-term.

B $x^2 - x - 30 = 0$ — Factoring; the equation is easily factorable.

C $6x^2 + 13x - 17 = 0$ — Quadratic formula, graphing; the equation cannot be factored.

D $x^2 - 5x + 3 = 0$ — Quadratic formula, completing the square, or graphing; the coefficient of the x^2-term is 1, but the equation cannot be factored.

E $-16x^2 - 50x + 21 = 0$ — Quadratic formula, graphing; the equation cannot be factored easily since the numbers are large.

Problem 4

Using the Discriminant

How many real-number solutions does $2x^2 - 3x = -5$ have?

Plan

Can you solve this problem another way?
Yes. You could actually solve the equation to find any solutions. However, you only need to know the number of solutions, so use the discriminant.

Think	Write
Write the equation in standard form.	$2x^2 - 3x + 5 = 0$
Evaluate the discriminant by substituting 2 for a, -3 for b, and 5 for c.	$b^2 - 4ac = (-3)^2 - 4(2)(5)$ $= -31$
Draw a conclusion.	Because the discriminant is negative, the equation has no real-number solutions.

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Use the quadratic formula to solve each equation.

1. $2x^2 + 5x + 3 = 0$ **2.** $5x^2 + 16x - 84 = 0$ **3.** $4x^2 + 7x - 15 = 0$

4. $3x^2 - 41x = -110$ **5.** $18x^2 - 45x - 50 = 0$ **6.** $3x^2 + 44x = -96$

7. Apply Mathematics (1)(A) A football player punts a ball. The path of the ball can be modeled by the equation $y = -0.004x^2 + x + 2.5$, where x is the horizontal distance, in feet, the ball travels and y is the height, in feet, of the ball. How far from the football player will the ball land? Round to the nearest tenth of a foot.

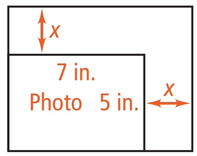
Use the quadratic formula to solve each equation. Round your answer to the nearest hundredth.

8. $x^2 + 8x + 11 = 0$ **9.** $5x^2 + 12x - 2 = 0$ **10.** $2x^2 - 16x = -25$

Select Techniques to Solve Problems (1)(C) Use techniques such as number sense and mental math to help you choose one or more methods of solving each equation. Explain your reasoning.

11. $x^2 + 4x - 15 = 0$ **12.** $9x^2 - 49 = 0$ **13.** $x^2 + 4x - 60 = 0$

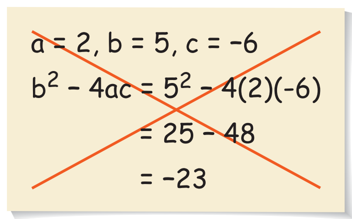
14. Apply Mathematics (1)(A) Your school wants to take out an ad in the paper congratulating the basketball team on a successful season, as shown at the right. The area of the photo will be half the area of the entire ad. What is the value of x?



15. Explain Mathematical Ideas (1)(G) You operate a dog-walking service. You have 50 customers per week when you charge \$14 per walk. For each \$1 decrease in your fee for walking a dog, you get 5 more customers per week. Can you ever earn \$750 in a week? Explain.

16. Explain Mathematical Ideas (1)(G) How can you use the discriminant to write a quadratic equation that has two solutions?

17. Explain Mathematical Ideas (1)(G) Describe and correct the error at the right that a student made in finding the discriminant of $2x^2 + 5x - 6 = 0$.



18. Find the discriminant and the solution of each equation in parts (a)–(c). If necessary, round to the nearest hundredth.

a. $x^2 - 6x + 5 = 0$

b. $x^2 + x - 20 = 0$

c. $2x^2 - 7x - 3 = 0$

d. Explain Mathematical Ideas (1)(G) When the discriminant is a perfect square, are the solutions rational or irrational? Explain.

19. **Explain Mathematical Ideas (1)(G)** The solutions of any quadratic equation $ax^2 + bx + c = 0$ are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

   a. Find a formula for the sum of the solutions.

   b. One solution of $2x^2 + 3x - 104 = 0$ is -8. Use the formula you found in part (a) to find the second solution.

For each condition given, tell whether $ax^2 + bx + c = 0$ will *always, sometimes,* or *never* have two solutions.

20. $b^2 < 4ac$ **21.** $b^2 = 0$ **22.** $ac < 0$

Find the number of real-number solutions of each equation.

23. $x^2 - 2x + 3 = 0$ **24.** $x^2 - 15 = 0$ **25.** $9x^2 + 12x + 4 = 0$

Use any method to solve each equation. If necessary, round your answer to the nearest hundredth.

26. $3w^2 = 48$ **27.** $3x^2 + 2x - 4 = 0$ **28.** $6g^2 - 18 = 0$

29. $3p^2 + 4p = 10$ **30.** $k^2 - 4k = -4$ **31.** $13r^2 - 117 = 0$

TEXAS End-of-Course PRACTICE

32. What are the approximate solutions of the equation $x^2 - 7x + 3 = 0$?

A. $-6.54, 0.46$ **B.** $-6.54, -0.46$ **C.** $-0.46, 6.54$ **D.** $0.46, 6.54$

33. Which of the following relations is a function?

F. $\{(1, 2), (3, 5), (1, 4), (2, 3)\}$ **H.** $\{(8, 2), (6, 3), (6, 11), (-8, 2)\}$

G. $\{(-5, 6), (0, 9), (-1, 2), (0, 6)\}$ **J.** $\{(-1, 3), (7, 3), (-7, 2), (4, 5)\}$

34. What equation do you get when you solve $3a - b = 2c$ for b?

A. $b = -3a + 2c$ **B.** $b = 3a - 2c$ **C.** $b = 3a + 2c$ **D.** $b = -3a - 2c$

35. What are the approximate solutions of the equation $\frac{1}{3}x^2 - \frac{5}{4}x + 1 = 0$? Use a graphing calculator.

F. 1.07, 2.77 **G.** 1.16, 2.59 **H.** 0.87, 10.38 **J.** 0.19, 16.01

36. Suppose the line through points $(n, 6)$ and $(1, 2)$ is parallel to the graph of $2x + y = 3$. Find the value of n. Show your work.

Topic 8 Review

TOPIC VOCABULARY

- axis of symmetry, *p. 327*
- completing the square, *p. 367*
- compression, *p. 340*
- discriminant, *p. 372*
- falling object model, *p. 327*
- linear factor, *p. 360*
- maximum, *p. 327*
- minimum, *p. 327*
- parabola, *p. 327*
- quadratic equation, *p. 350*
- quadratic formula, *p. 372*
- quadratic function, *p. 326*
- quadratic parent function, *p. 327*
- root of an equation, *p. 350*
- standard form of a quadratic equation, *p. 350*
- standard form of a quadratic function, *p. 326*
- stretch, *p. 340*
- vertex, *p. 327*
- vertex form, *p. 346*
- vertical motion model, *p. 333*
- zero of a function, *p. 350*
- Zero-Product Property, *p. 356*

Check Your Understanding

Choose the vocabulary term that correctly completes the sentence.

1. The U-shaped graph of a quadratic function is a(n) __?__.
2. The __?__ can be used to determine the number of real-number solutions of a quadratic equation.
3. The line that divides a parabola in half is the __?__.
4. The __?__ of a parabola is the point at which the parabola intersects the axis of symmetry.

8-1 and 8-2 Graphing Quadratic Functions

Quick Review

A function of the form $y = ax^2 + bx + c$, where $a \neq 0$, is a **quadratic function**. Its graph is a **parabola**. The **axis of symmetry** of a parabola divides it into two matching halves. The **vertex** of a parabola is the point at which the parabola intersects the axis of symmetry.

Example

What is the vertex of the graph of $y = x^2 + 6x - 2$?

The x-coordinate of the vertex is given by $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$$

Find the y-coordinate of the vertex.

$y = (-3)^2 + 6(-3) - 2$ Substitute -3 for x.

$y = -11$ Simplify.

The vertex is $(-3, -11)$.

Exercises

Graph each function. Label the axis of symmetry and the vertex.

5. $y = \frac{2}{3}x^2$
6. $y = -x^2 + 1$
7. $y = x^2 - 4$
8. $y = 5x^2 + 8$
9. $y = -\frac{1}{2}x^2 + 4x + 1$
10. $y = -2x^2 - 3x + 10$
11. $y = \frac{1}{2}x^2 + 2x - 3$
12. $y = 3x^2 + x - 5$

Give an example of a quadratic function that matches each description.

13. Its graph opens downward.
14. The vertex of its graph is at the origin.
15. Its graph opens upward.
16. Its graph is wider than the graph of $y = x^2$.

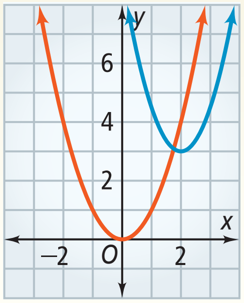
8-3 Transformations of Quadratic Functions

Quick Review

You can graph a quadratic function by identifying its **transformation** from the parent function $y = x^2$. Examples of such transformations would be a reflection, a **compression,** a **stretch,** and a translation.

Example

What steps transform the graph of $f(x) = x^2$ to the graph of $g(x) = 1.5(x - 2)^2 + 3$?



Because $|a| > 1$, the graph stretches vertically. The vertex of $f(x)$ is $(0, 0)$ and the vertex of $g(x)$ is $(2, 3)$. So the transformation includes a translation, 2 units to the right and 3 units up.

Exercises

Identify each function as a translation, compression, stretch, or reflection of the parent function, $f(x) = x^2$.

17. $f(x) = x^2 - 3$

18. $f(x) = 2x^2$

19. $f(x) = -x^2$

20. $f(x) = (x + 5)^2$

What steps transform the graph of $y = x^2$ to the graph of each of the functions below.

21. $y = (x - 3)^2 + 1$

22. $y = -2x^2$

Sketch both functions on the same coordinate grid.

23. $f(x) = x^2$; $g(x) = (x - 3)^2 - 2$

24. $f(x) = x^2$; $g(x) = 2x^2 + 3$

25. $f(x) = (x - 1)^2$; $g(x) = -\frac{1}{3}(x - 1)^2$

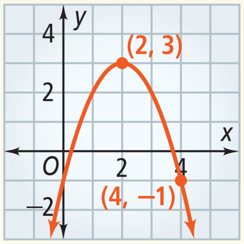
8-4 Vertex Form of a Quadratic Function

Quick Review

The graph of a quadratic function is a parabola. You can use the **vertex form** of a quadratic function, which is $y = a(x - h)^2 + k$, to identify the vertex and shape of the parabola. The vertex is (h, k).
If $a > 0$, k is the **minimum value** of the function. If $a < 0$, k is the **maximum value** of the function. The **axis of symmetry** is given by $x = h$.

Example

What is the equation for the parabola shown in the graph?



The vertex is $(2, 3)$, so the equation is $f(x) = a(x - 2)^2 + 3$. Find a by substituting the coordinates of another point on the parabola, such as $(4, -1)$.

$$-1 = a(4 - 2)^2 + 3$$
$$-1 = a(2)^2 + 3$$
$$-4 = 4a$$
$$a = -1$$

The equation for the parabola is $f(x) = -(x - 2)^2 + 3$.

Exercises

Find the vertex of each parabola, and identify its shape as opening upward or downward.

26. $f(x) = -x^2 + 10$

27. $f(x) = 2(x - 1)^2 + 5$

28. $f(x) = -\frac{1}{4}(x + 6)^2 - \frac{1}{8}$

29. $f(x) = 3(x - 3.5)^2 - 2.8$

Use the description of the parabola to write its equation in vertex form.

30. a parabola with vertex $(1, 2)$; passes through $(2, 3)$

31. a parabola with vertex $(0, -3)$; passes through $(3, 0)$

32. a parabola with vertex $(-1, 4)$; passes through $(-2, 3)$

33. A classmate says that exactly two parabolas have a vertex at $(3, 4)$—one that opens upward and one that opens downward. Do you agree? Explain.

34. Let $f(x) = -2(x - 1)^2 + 4$. Is there a value of x such that $f(x) = 5$? Explain.

8-5 and 8-6 Solving Quadratic Equations

Quick Review

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$. Quadratic equations can have two, one, or no real-number solutions. You can solve a quadratic equation by graphing the related function and finding the x-intercepts. Some quadratic equations can also be solved using square roots. If the left side of $ax^2 + bx + c = 0$ can be factored, you can use the **Zero-Product Property** to solve the equation.

Example

What are the solutions of $2x^2 - 72 = 0$?

$2x^2 - 72 = 0$	
$2x^2 = 72$	Add 72 to each side.
$x^2 = 36$	Divide each side by 2.
$x = \pm\sqrt{36}$	Find the square roots of each side.
$x = \pm 6$	Simplify.

Exercises

Solve each equation. If the equation has no real-number solution, write *no solution.*

35. $6(x^2 - 2) = 12$

36. $-5m^2 = -125$

37. $9(w^2 + 1) = 9$

38. $3r^2 + 27 = 0$

39. $4 = 9k^2$

40. $4n^2 = 64$

Solve by factoring.

41. $x^2 + 7x + 12 = 0$

42. $5x^2 - 10x = 0$

43. $2x^2 - 9x = x^2 - 20$

44. $2x^2 + 5x = 3$

45. $3x^2 - 5x = -3x^2 + 6$

46. $x^2 - 5x + 4 = 0$

47. The area of a circle A is given by the formula $A = \pi r^2$, where r is the radius of the circle. Find the radius of a circle with area 16 in.^{2}. Round to the nearest tenth of an inch.

8-7 Writing Quadratic Functions

Quick Review

The solutions of the **quadratic equation** $ax^2 + bx + c = 0$ are also the **zeros** of the related **quadratic function** $f(x) = ax^2 + bx + c$. The zeros are also the x-intercepts of the graph of the quadratic function.

Example

What are the solutions of the quadratic equation $x^2 + 17x - 18 = 0$ and the zeros of the related quadratic function?

Factor the quadratic equation $x^2 + 17x - 18 = 0$ and find its solutions.

$$x^2 + 17x - 18 = 0$$

$$(x + 18)(x - 1) = 0$$

Since $x + 18 = 0$ or $x - 1 = 0$, the solutions of the quadratic equation are -18 and 1.

The solutions of a quadratic equation are also the zeros of the related quadratic function. The zeros of $f(x) = x^2 + 17x - 18$ are -18 and 1.

Exercises

Identify the zeros of the quadratic function.

48. $y = x^2 - 5x - 36$

49. $y = x^2 + 6x + 8$

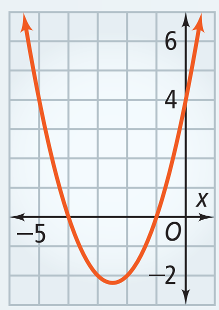
Use the given solutions of a quadratic equation to write the equation of a related quadratic function in standard form.

50. $-6, 3$

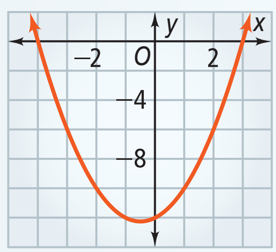
51. $7, -4$

Write a quadratic function in standard form for the parabola shown.

52.



53.



8-8 Completing the Square

Quick Review

You can solve any quadratic equation by writing it in the form $x^2 + bx = c$, **completing the square,** and finding the square roots of each side of the equation.

Example

What are the solutions of $x^2 + 8x = 513$?

$x^2 + 8x + 16 = 513 + 16$	Add $\left(\frac{8}{2}\right)^2$, or 16, to each side.
$(x + 4)^2 = 529$	Write $x^2 + 8x + 16$ as a square.
$x + 4 = \pm\sqrt{529}$	Find the square roots.
$x + 4 = \pm 23$	Simplify.
$x + 4 = 23$ or $x + 4 = -23$	Write as two equations.
$x = 19$ or $x = -27$	Solve for x.

Exercises

Solve each equation by completing the square. If necessary, round to the nearest hundredth.

54. $x^2 + 6x - 5 = 0$

55. $x^2 = 3x - 1$

56. $2x^2 + 7x = -6$

57. $x^2 + 10x = -8$

58. $4x^2 - 8x = 24$

59. $x^2 - 14x + 16 = 0$

60. **Apply Mathematics (1)(A)** You are planning a rectangular patio with length that is 7 ft less than three times its width. The area of the patio is 120 ft^2. What are the dimensions of the patio to the nearest tenth?

61. **Apply Mathematics (1)(A)** You are designing a rectangular birthday card for a friend. You want the card's length to be 1 in. more than twice the card's width. The area of the card is 88 $in.^2$. What are the dimensions of the card to the nearest tenth?

8-9 The Quadratic Formula and the Discriminant

Quick Review

You can solve the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, by using the **quadratic formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The **discriminant** is $b^2 - 4ac$. The discriminant tells you how many real-number solutions the equation has.

Example

How many real-number solutions does the equation $x^2 + 3 = 2x$ have?

$x^2 - 2x + 3 = 0$	Write in standard form.
$b^2 - 4ac = (-2)^2 - 4(1)(3)$	Evaluate discriminant.
$= -8$	Simplify.

Because the discriminant is negative, the equation has no real-number solutions.

Exercises

Find the number of real-number solutions of each equation.

62. $x^2 + 7x - 10 = 3$

63. $3x^2 - 2 = 5x$

Solve each equation using the quadratic formula. Round to the nearest hundredth.

64. $4x^2 + 3x - 8 = 0$

65. $2x^2 - 3x = 20$

66. $-x^2 + 8x + 4 = 5$

67. $64x^2 + 12x - 1 = 0$

Solve each equation using any method. Explain why you chose the method you used.

68. $5x^2 - 10 = x^2 + 90$

69. $x^2 - 6x + 9 = 0$

70. A ball is thrown into the air. The height h, in feet, of the ball can be modeled by the equation $h = -16t^2 + 20t + 6$, where t is the time, in seconds, the ball is in the air. When will the ball hit the ground?

Topic 8 TEKS Cumulative Practice

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. Which expression is equivalent to $(m^4n^{-1})(mp^2)(np^{-6})$?

   A. m^5p^{-4}

   B. m^5np^{-4}

   C. m^3np^4

   D. m^4np^{-12}

2. How would the graph of the function $y = x^2 - 5$ change if the function became $y = x^2 + 2$?

   F. The graph would shift 2 units down.

   G. The graph would shift 3 units up.

   H. The graph would shift 7 units up.

   J. The graph would shift 10 units down.

3. Which of the following is a function rule for the sequence 3, 8, 13, 18, 23, . . . ?

   A. $A(n) = 5 + (n - 1)(3)$

   B. $A(n) = 3 + (n - 1)(5)$

   C. $A(n) = 1 + (n - 1)(5)$

   D. $A(n) = 1 + (n - 1)(3)$

4. What are the solutions of $4v + 18 \geq 6v + 10$?

   F. $v \leq 4$

   G. $v \geq 4$

   H. $v < 4$

   J. $v > 4$

5. What are the solutions of $2x^2 - 11x + 5 = 0$?

   A. 2, 5

   B. −5, −0.5

   C. 0.5, 5

   D. −5, −2

6. The difference of Ann's and Jay's heights is half of Jay's height. If Ann is taller than Jay, which equation represents Ann's height a in terms of Jay's height j?

   F. $a = \frac{1}{2}j - j$

   G. $a = j - \frac{1}{2}j$

   H. $a = \frac{1}{2}j + j$

   J. $a = 2j - j$

7. Keisha's grandmother gave her a doll that she paid \$6 for 60 years ago. The doll's current value is \$96. Its value doubles every 15 years. What will the doll be worth in 60 years?

   A. \$570 C. \$1536

   B. \$768 D. \$3072

8. The length of a rectangle is represented by the expression $n - 3$. The width of the rectangle is represented by the expression $4n + 5$. Which expression represents the area of the rectangle?

   F. $4n^2 - 12n - 15$ H. $-3n - 15$

   G. $4n^2 - 17n - 15$ J. $4n^2 - 7n - 15$

9. Which expression is equivalent to $\left(\frac{x^4y^{-2}}{z^3}\right)^{-3}$?

   A. $\frac{y^6z^3}{x^{12}}$ C. $\frac{y^6z^9}{x^{12}}$

   B. $\frac{y^6}{x^{12}z^9}$ D. $\frac{y^6}{x^{12}z^3}$

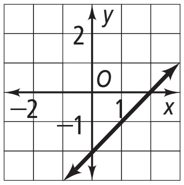
10. The table shows the number of volunteers v needed based on the number of children c who will go on a field trip. Which equation best represents the relationship between the number of volunteers and the number of children?

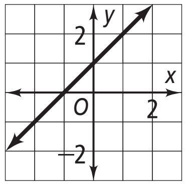
c	v
20	6
25	7
30	8
35	9

   F. $v = 0.25c + 10$ H. $v = 0.2c + 2$

   G. $v = 5c - 10$ J. $v = 4c + 2$

11. Which graph shows a line that is parallel to the line with equation $4x - 8y = 10$?

A.


C.


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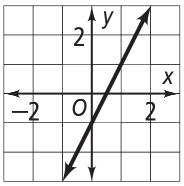

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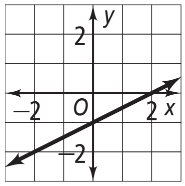
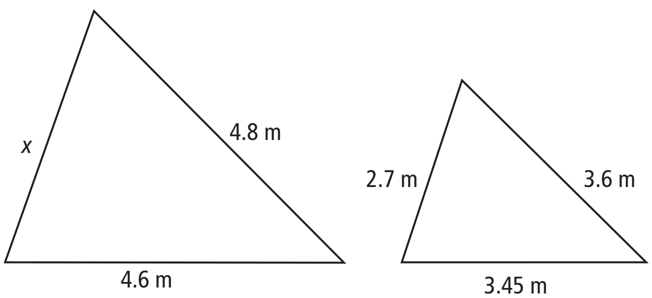


Gridded Response

12. Alan is tiling a 6 ft-by-8 ft rectangular floor with square tiles that measure 4 in. on each side. How many tiles does Alan need to cover the floor?

13. A library is having a used book sale. All hardcover books have the same price and all softcover books have the same price. You buy 4 hardcover books and 2 softcover books for \$24. Your friend buys 3 hardcover books and 3 softcover books for \$21. What is the cost in dollars of a hardcover book?

14. The two triangles below are similar.



What is the length, in meters, of the side labeled x?

15. One model of a jumbo jet is approximately 230 ft long and has a wingspan of 195 ft. If a scale model of the plane is about 40 cm long, what is the model's wingspan in centimeters? Round to the nearest tenth.

Constructed Response

16. How many real-number solutions does the quadratic equation $2x^2 + 7x + 9 = 0$ have?

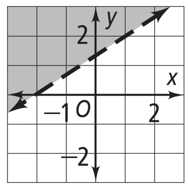
17. Terry says that a quadratic equation has two real solutions. Is this statement *always*, *sometimes*, or *never* true? Give two examples to support your answer.

18. An equation of line p is $y = 4x - 3$. Line n is perpendicular to line p and contains the point $(8, -1)$. What is an equation of line n? Show your work.

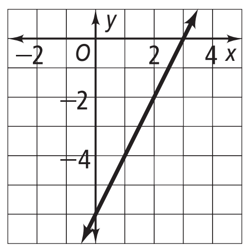
19. A caterer charges a \$50 fee plus \$12 per person for an event. Write a function to represent the total cost C for n people.

20. Factor the expression $3x^2 + 23x + 14$.

21. What is an inequality that represents the graph?



22. What is an equation of the line?



23. A system of equations is shown below.

$$y = 2x + 5$$
$$y = -x + 11$$

a. Graph the equations in the same coordinate plane.

b. What is the point of intersection of the two graphs?

24. Graph the function $y = 2x^2$. Make a table of values. What are the domain and range?

Topic 8

Lesson 8-1

Identify the domain and range of each function.

1. $y = 3x^2$
2. $y = -4x^2$
3. $y = -0.5x^2$
4. $y = 2x^2 + 5$
5. $y = -0.3x^2 - 7$
6. $y = -5x^2 + 0.8$

7. Water from melting snow drips from a roof at a height of 40 ft. The function $h = -16t^2 + 40$ gives the approximate height h in feet of a drop of water t seconds after it falls. Graph the function.

Lesson 8-2

Find the axis of symmetry and the coordinates of the vertex of the graph of each function.

8. $y = 3x^2$
9. $y = -5x^2 + x + 4$
10. $y = 0.5x^2 - 3$
11. $y = -x^2 + 2x + 1$
12. $y = 3x^2 + 6x$
13. $y = x^2 - 8x$

Use the axis of symmetry, vertex, and *y*-intercept to graph each function.

14. $y = x^2 - 4$
15. $y = 2x^2 + x$
16. $y = x^2 + x - 2$

The formula $h = -16t^2 + vt + c$ describes the height of an object thrown into the air, where h is the height, t is the time in seconds, v is the initial velocity, and c is the initial height. Use the formula to answer each question.

17. A football is thrown with an upward velocity of 15 ft/s from an initial height of 5 feet. How long will it take for the football to reach its maximum height?
18. A ball is thrown from the top of a 50-ft building with an upward velocity of 24 ft/s. When will it reach its maximum height? How far above the ground will it be?

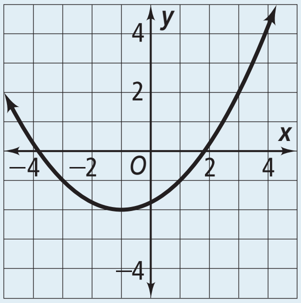
Lesson 8-3

19. Graph $g(x) = \frac{1}{4}x^2$ and $h(x) = \left(\frac{1}{4}x\right)^2$ on the same coordinate grid. Describe how each function is a stretch, compression, reflection, or translation of $f(x) = x^2$.
20. Graph $g(x) = x^2 + 2$ and $h(x) = (x + 2)^2$ on the same coordinate grid, and describe how each function is a stretch, compression, reflection, or translation of $f(x) = x^2$.
21. Describe the steps that transform the graph of $y = x^2$ to $y = (x - 2)^2 + 1$.

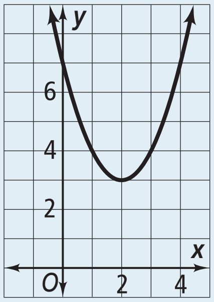
Lesson 8-4

For each graph, write a quadratic function in vertex form and standard form.

22.



23.



Find the vertex, the axis of symmetry, whether the parabola opens upward ordownward, and the domain and range of the function.

24. $y = 2(x - 1)^2 + 1$

25. $y = \frac{1}{3}(x + 3)^2 - 2$

26. $y = -4(x + 2)^2 - 3$

Lessons 8-5 and 8-6

Solve each equation. If the equation has no solution, write *no solution*.

27. $x^2 = 36$

28. $x^2 + x - 2 = 0$

29. $c^2 - 100 = 0$

30. $9d^2 = 25$

31. $h^2 + 4 = 0$

32. $3x^2 = 27$

Model each problem with a quadratic equation. Then solve. If necessary, round to the nearest tenth.

33. Find the radius of a circular lid with an area of 12 in.^2.

34. Find the diameter of a circular pond with an area of 300 m^2.

Lesson 8-7

The solutions of a quadratic equation are given. Write the related quadratic functions in factored form.

35. −3, 2

36. 4, 1

37. −1, −2

38. 6, 8

Lessons 8-8 and 8-9

Solve each equation. If the equation has no solution, write *no solution*.

39. $x^2 + 6x - 2 = 0$

40. $x^2 - 5x = 7$

41. $3x^2 + x + 5 = 0$

42. $-3x^2 + x - 7 = 0$

43. $x^2 + 5x + 6 = 0$

44. $d^2 + 2d + 10 = 2d + 100$

Find the number of real-numbered solutions of each equation.

45. $3x^2 + 4x - 7 = 0$

46. $5x^2 - 4x = -6$

47. $x^2 - 20x + 101 = 1$

Solve by completing the square.

48. A rectangular patio has a length of $x + 6$ m, a width of $x + 8$ m, and a total area of 400 m^2. Find the dimensions to the nearest tenth.